THE

THEORY OF STRAINS.

JOHN H. DIEDRICHS

Feb. 23, 1918.

~~500~~
50 ¢

THE

THEORY OF STRAINS.

A COMPENDIUM FOR THE

CALCULATION AND CONSTRUCTION OF BRIDGES, ROOFS AND CRANES,

WITH THE APPLICATION OF

TRIGONOMETRICAL NOTES.

CONTAINING

THE MOST COMPREHENSIVE INFORMATION IN REGARD TO THE RESULTING STRAINS FOR A PERMANENT LOAD, AS ALSO FOR A COMBINED (PERMANENT AND ROLLING) LOAD.

IN TWO SECTIONS.

ADAPTED TO THE REQUIREMENTS OF THE PRESENT TIME.

BY

JOHN H. DIEDRICHS,

CIVIL AND MECHANICAL ENGINEER.

ILLUSTRATED BY NUMEROUS PLATES AND DIAGRAMS.

BALTIMORE:
SCHMIDT & TROWE.
1871.

Westcott & Thomson,
Stereotypers, Philada.

INSCRIBED TO

WENDEL BOLLMAN, Esq.,

IN TESTIMONY OF HIS INTEREST IN SCIENCE AND ART.

CONTENTS.

PAGE

PREFACE ... 7

EXPLANATION OF CHARACTERS USED IN THE CALCULATIONS ... 11

SECTION I.

A. INTRODUCTION ... 13

I. THE LEVER ... 15

II. SUSPENDED WEIGHTS AND THE RESULTING STRAINS ... 18

III. TRUSSES WITH SINGLE AND EQUALLY-DISTRIBUTED LOAD ... 21

Suspension Truss Bridge ... 23

Suspension Bridge ... 27

B. ROOF CONSTRUCTION ... 31

C. SEMI-GIRDERS ... 43

I. SEMI-GIRDERS LOADED AT THE EXTREMITY ... 43

Cranes ... 44

II. SEMI-GIRDERS LOADED AT EACH APEX ... 45

D. GIRDERS WITH PARALLEL TOP AND BOTTOM FLANGES 48

I. STRAIN IN DIAGONALS AND VERTICALS ... 48

II. STRAIN IN FLANGES ... 51

III. TRANSFORMATIONS ... 53

IV. GENERAL REMARKS ... 53

Directions for the Calculation of Complex Systems ... 54

E. COMPARATIVE TABLES OF RESULTING STRAINS FOR A PERMANENT LOAD ... 56

I. SYSTEM OF RIGHT-ANGLED TRIANGLES ... 56

II. SYSTEM OF ISOSCELES BRACING ... 58

SECTION II.

Girders Calculated for Combined (Permanent and Rolling) Load.

PAGE

A. GIRDERS WITH PARALLEL TOP AND BOTTOM FLANGES 59

I. The Right-angled System 59

II. Isosceles Bracing 66

1. Triangular Truss 66

Calculation of Strains y and u in Diagonals 67

2. Isometrical Truss 69

a. Calculation of Diagonals 70

b. Calculation of Top and Bottom Chords (Flanges) 72

B. CAMBER IN TRUSSES, WITH PARALLEL TOP AND BOTTOM CHORDS 73

Tables containing the Length of Arches for Degrees, Minutes and Seconds, for a Radius as Unit 77

C. PARABOLIC GIRDER OF 48 FEET, OR 16 METRES, SPAN 80

D. THE ARCHED TRUSS 86

Calculation of Strain y in the Diagonals 88

Calculation of Strain in the Verticals V 90

Transformations 93

E. THRUST CONSTRUCTION 94

I. Girder 20 feet in Length (with a Single Weight at the Centre).

II. Girder 72 feet in Length (calculated for Permanent and Rolling Load).

Definition of Strain x in the Horizontal Flanges 97

Definition of Strain y in the Diagonals 98

Calculation of the Tensile Strains z in the Lower Flanges 99

Calculation of the Verticals u 101

III. Calculation of a Truss sustaining a Dome 102

Calculation of Strains x of the Outside Arch 104

Calculation of Strains z of the Inside Arch 105

Calculation of the Diagonals y 106

PREFACE.

THE want of a compact, universal and popular treatise on the construction of Roofs and Bridges—especially one treating of the influence of a variable load—and the unsatisfactory essays of different authors on the subject, induced me to prepare the following work.

Bridge-building has been and always will be an important branch of industry, not only to engineers, but also to the masses for the purposes of travel and trade, and, as Colonel Merrill in his recent essay on Bridge-building remarks, "important to railroad companies on account of the large amount of capital invested in their construction."

Bridge literature has often been used by rival parties for the purpose of advancing their own private interests, their motive being competition. Imposing upon the faith and credulity of those whom they pretend to serve, there is no guarantee that worthless structures will not be erected.

Thoroughly independent of any such motive, my aim is to give, especially to bridge-builders and to engineers and architects, the results of my investigation on the subject of calculating strains, in order that capitalists and the public may be benefited and protected.

These calculations will also enable those who have but a limited knowledge of mathematics to acquire the necessary information. For this reason special attention is paid to the arrangement of the work, the whole being made as plain and simple as possible, in order to meet the wants of the common mechanic as well as the experienced engineer.

Though there are many valuable treatises of this kind, there has as yet been no work published serviceable to the degree desired by the practical builder or mechanic—most of the dissertations being too theoretical and hard to comprehend by one not versed in the higher mathematics; and some are so arranged that a clear understanding of the calculations is very difficult.*

The most valuable work in the language is doubtless Mr. Stoney's "Theory of Strains," though the Method of Moments is not developed to that degree which I think necessary for the practical man.

We owe to the renowned German engineers Ritter and Von Kaven the universal application of this *Method* in the work entitled "Dach und Brücken Constructionen," in which it is fully explained by examples and illustrated by diagrams, these being often carelessly neglected in other works.

The above-mentioned work served me very much in the arrangement of this, which I hope will be kindly received.

The work being expressly prepared, as aforesaid, for the use of beginners in the study of mathematics, as well as for the

---

* As an exception, may be named Mr. Shreve's brief but popular treatise in Van Nostrand's "Engineering Magazine," No. xx., August, 1870; Vol. III.

more advanced practical engineer, it will enable them, after a short perusal, to acquire all the necessary information, for which even the trigonometrical notes accompanying the general results are not really required.

The higher classes of colleges and other institutions of learning will find the work very valuable.

On account of the expense, an intended Appendix, containing a rational and concise investigation on "The Strength of Materials," had to be dispensed with; yet I hope with this volume to gratify not only the desire of friends, but to be able with great satisfaction to assist engineers in the pursuit of their high and noble calling.

THE AUTHOR.

OCTOBER, 1870.

EXPLANATION OF CHARACTERS USED IN THE CALCULATIONS.

$=$ Equal, or the sign of equality.

$+$ Plus, or the sign of addition; also, the symbol of positive (tensile) strain.

$-$ Minus, or the sign of subtraction; also, the symbol of negative (compressive) strain.

$\times$ *or* . Sign of multiplication.

: *or* $\div$ Sign of division.

, Sign of decimals; also, of thousands.

∞ Sign of infinite.

$<$ Sign of angle, signified by the Grecian cyphers α, β, γ, δ, ε.

^{2} Sign of square of a number.

$\sqrt{\ }$ Sign of square root of a number.

$^\circ$ Sign of degrees.

$'$ Sign of minutes; also, of feet.

$''$ Sign of seconds; also, of inches.

( ) *or* [ ] Brackets, to enclose the mathematical expression bound to the same operation.

π The number 3, 14, or periphery for a unit of the diameter.

R Right angle, or 90°.

$\perp$ Vertical.

THE THEORY OF STRAINS.

SECTION I.

A. INTRODUCTION.

To enable the student to comprehend the work, and have a thorough knowledge of certain conditions and examples without studying the whole, it is necessary for him to understand the arrangement of the following pages.

On the first few pages and the appertaining figures at the close of the chapter is found a short description of the lever in its different appliances, the application being only a key to the calculations of strains which follow.

The trigonometrical notes are in many cases almost superfluous. Still, it may be advantageous in this way to accustom the reader to their use. The "Suspended Weights and Resulting Strains" are developed by the parallelogram of forces, and for a plain illustration the results are appended to the figures, which will also be observed on figures of "Trusses with Single and Distributed Load."

In the "Suspended Weights and Resulting Strains" a more elaborate calculation was thought necessary, and therefore an Introduction to the calculation by the "Method of Moments" may be found in its proper place.

This Introduction presents the beginner with a clear and comprehensive knowledge of the formation of Moments; and the equations for Figs. 20, 21, etc., explain the equilibrium of force and leverage.

At the close of this chapter is found the explanation of maximum compressive and tensile strain in the top and bottom chords of a parallel-flanged truss or girder.

On "Roof Construction" (B), Plates 6 to 11, remarks are un-

necessary. The builder can with ease find from the figures a system to suit his purpose. (See also "Arched Trusses," D, Section II.)

On "Semi-Girders" (C) the calculation of strains is treated in the way heretofore generally known (determining from the centre toward the abutment), after which the "Method of Moments" is applied to the same example, followed by a more elaborate explanation of the principles of Moments on the crane skeleton (Plate 14), whose single members are altogether divergent.

The thorough calculation of a truss with horizontal top and bottom flanges (right-angled system D, Sect. I.), with the resulting strains for a system of braces, all of which are inclined in the same direction, shows how easily by transformation of the strains a system of bracing just reversed can be formed. (Comp. D, Sect. II.)

From the comparative tables, E, I., II. (Plate 18, 19) those who are not mathematicians can find, by a little study (for an assumed load "W"—a variable load not considered), the strains in flanges, braces and ties. (See D, IV., Sect. I.)

The progress of panels, and by this the increase of stress in the different members, are *ad libitum* to be extended (are optional).

Where no composite strains appear, in the skeletons double lines make the compressive (—) strains more obvious, the tensile (+) strains being always represented by single lines. The assumed load in the calculations is equally divided on the apexes; but in general some attention may be paid to a peculiar load—say from a single locomotive—at a certain apex, this being observed in examples on "Suspension Truss" (III., Sect. I.).

The calculations in Sect. II. with regard to the influence of a variable load are more difficult to understand. Still, by the results of strains in the skeletons it is easy enough to form an idea about this matter, and to see the importance of counter-bracing or tying at centre of railroad-bridge trusses.

What experience and observation have already taught to the practical railroad man is here *fully* shown by figures.

Information is given on parallel-flanged trusses for the so-called "Camber in Bridges" at B, Sect. II., which to many builders has heretofore been only a matter of experiment.

Yet it is to be remarked that for the calculations E, Sect. II., Plates

29 to 34, a variation in
will be perceived, the
being the horizontal
vertex (centre), an
ments.

Farther expl
referred to.

The calcul
nections at
relieving th
strain. T
ture are

Pla
Fi

fig. 1b

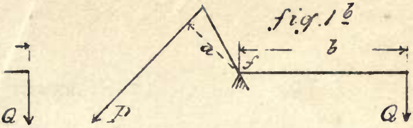


fig. 1c

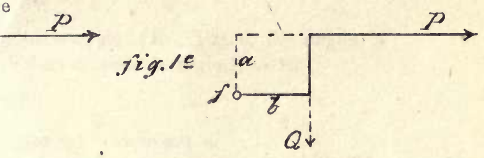


fig. 2.

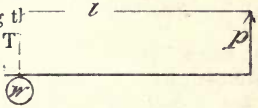


fig. 3.

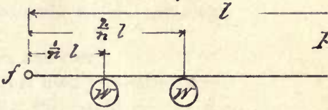


fig. 4.

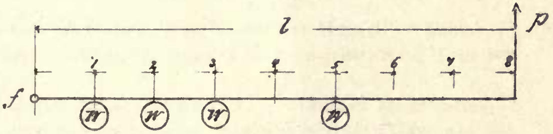


fig. 5

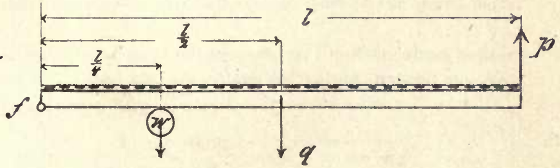


fig. 6.

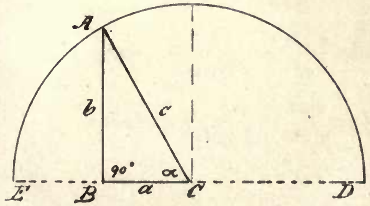
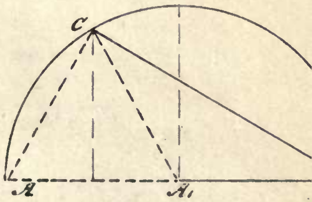


fig. 7.



necessary. The builder can with ease from the fulcrum equal system to suit his purpose. (See also "A tion II.)

On "Semi-Girders" (C) the calculation of the way heretofore generally known (determini toward the abutment), after which the "Methoa applied to the same example, followed by a more e nation of the principles of Moments on the crane s. 14), whose single members are altogether divergent. is divided in

The thorough calculation of a truss with horizont bottom flanges (right-angled system D, Sect. I.), with th strains for a system of braces, all of which are inclin same direction, shows how easily by transformation of th a system of bracing just reversed can be formed. (Co Sect. II.)

From the comparative tables, E, I., II. (Plate 18, 19) thos are not mathematicians can find, by a little study (for an assu load "W"—a variable load not considered), the strains in flangad, braces and ties. (See D, IV., Sect. I.)

The progress of panels, and by this the increase of stress in th different members, are *ad libitum* to be extended (are optional).

Where no composite strains appear, in the skeletons double lines make the compressive (—) strains more obvious, the tensile (+) strains being always represented by single lines. The assumed load in the calculations is equally divided on the apexes; but in general some attention may be paid to a peculiar load—say from a single locomotive—at a certain apex, this being observed in examples on "Suspension Truss" (III., Sect. I.).

The calculations in Sect. II. with regard to the influence of a variable load are more difficult to understand. Still, by the results of strains in the skeletons it is easy enough to form an idea about this matter, and to see the importance of counter-bracing or tying at centre of railroad-bridge trusses.

What experience and observation have already taught to the practical railroad man is here *fully* shown by figures.

Information is given on parallel-flanged trusses for the so-called "Camber in Bridges" at B, Sect. II., which to many builders has heretofore been only a matter of experiment.

Yet it is to be remarked that for the calculations E, Sect. II., Plates

fig. 1a

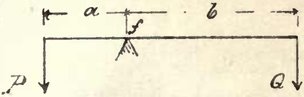


fig. 1b

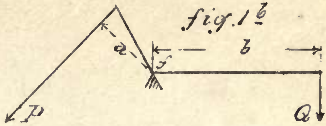


fig. 1c

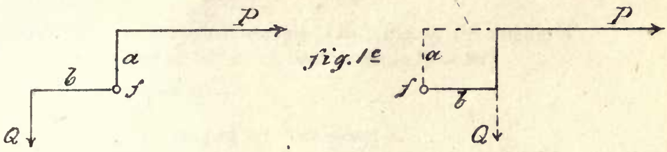


fig. 2.

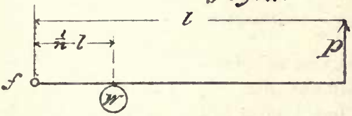


fig. 3.

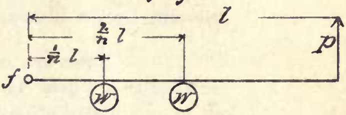


fig. 4.

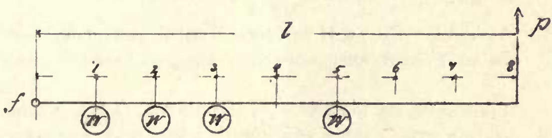


fig. 5

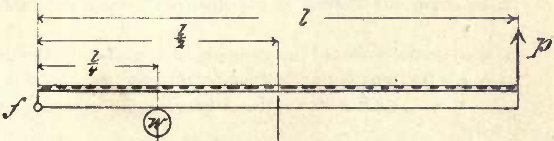


fig. 6.

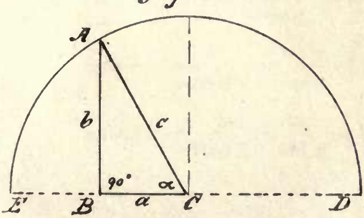
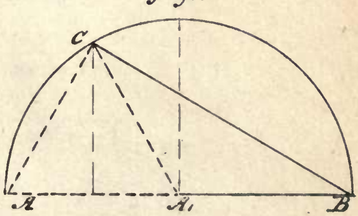


fig. 7.



In every triangle the sum of enclosed angles,

$$2\,R. = 2 \times 90°,$$

6.] so in the right angled triangle ABC, Fig. 6, the angles A and C together $= 90° = R$, because angle $B = 90°$.

In Trigonometry we say—

$\frac{b}{c} = \text{sine } \propto$; $\frac{b}{a} = \text{tangent } \propto$; $\frac{c}{a} = \text{secant } \propto$.

$\frac{a}{c} = \text{cosine } \propto$; $\frac{a}{b} = \text{cotangent } \propto$; $\frac{c}{b} = \text{cosecant } \propto$; or, contracted, sin., cos., tang. or tg., cotang. or cotg., sec. and cosec.

For a radius, AC, as a unit, the line AB simply is called sine; the central distance, BC, cosine; and BE the versed sine of the angle $\propto$.

For certain angles $\propto$ the relation $\frac{b}{c}, \frac{b}{a}, \frac{c}{a}$, etc., have certain and distinct numerical values. (See Haslett's or Haswell's "Tables of natural sines, cosines, tangents and cotangents from 1 to 90 degrees.")

Each triangle, $AB.C$, A_1BC (Fig. 7), consists of six members—*i. e.*, three sides and three angles, from which always three are dependent on the rest; therefore, when three out of these six members are known, we can construct, or with more exactness we can calculate, the others, provided one at least of the given parts is a side.

For the transformation of trigonometrical functions, short notices in the form of a table, also the numerical values (natural sin, cos, etc.) of the principal angles, may be serviceable, viz.:

$$\sin x = \frac{1}{\operatorname{cosec} x} = \frac{\operatorname{tang} x}{\sec x} = \sqrt{1 - \cos^2 x}.$$

$$\cos x = \frac{1}{\sec x} = \frac{\operatorname{cotang} x}{\operatorname{cosec} x} = \sqrt{1 - \sin^2 x}.$$

$$\operatorname{tang} x = \frac{1}{\operatorname{cotang} x} = \frac{\sin x}{\cos x} = \sqrt{\sec^2 - 1}.$$

$$\operatorname{cotang} x = \frac{1}{\operatorname{tang} x} = \frac{\cos x}{\sin x} = \sqrt{\operatorname{cosec}^2 x - 1}.$$

$$\sec x = \frac{1}{\cos x} = \frac{\operatorname{cosec} x}{\operatorname{cotang} x} = \sqrt{1 + \operatorname{tang}^2 x}.$$

$$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{\sec x}{\operatorname{tang} x} = \sqrt{1 + \operatorname{cotang}^2 x}.$$

DEGREES.	SIN.	COS.	TANG.	COTANG.
0	0	1	0	∞
15	0,258	0,965	0,267	3,732
20	0,342	0,939	0,363	2,747
25	0,422	0,906	0,466	2,144
30	$\frac{1}{2} = 0{,}5$	$\frac{1}{2}\sqrt{3} = 0{,}866$	$\frac{1}{3}\sqrt{3} = 0{,}577$	$\sqrt{3} = 1{,}732$
40	0,642	0,766	0,839	1,191
45	$\frac{1}{2}\sqrt{2} = 0{,}707$	$\frac{1}{2}\sqrt{2} = 0{,}707$	1,00	1,00
50	0,766	0,642	1,191	0,839
60	$\frac{1}{2}\sqrt{3} = 0{,}866$	$\frac{1}{2} = 0{,}5$	$\sqrt{3} = 1{,}732$	$\frac{1}{3}\sqrt{3} = 0{,}577$
65	0,906	0,422	2,144	0,466
75	0,965	0,258	3,732	0,267
$R = 90$	$+1$	0	$+\infty$	0
$2R = 180$	0	-1	0	$-\infty$
$3R = 270$	-1	0	$-\infty$	0
$4R = 360$	0	$+1$	0	$+\infty$

$\sin (R - x) = {}^{+}\cos x$, and $\cos (R - x) = {}^{+}\sin x$.
$\sin (R + x) = {}^{+}\cos x$, and $\cos (R + x) = {}^{-}\sin x$.

II. *SUSPENDED WEIGHTS AND THE RESULTING STRAINS.*

Plate 2, Fig. 8a, " 8b.] In Fig. 8b, when $W = 5000$ lbs, $ab = be = 10$, $bc = 8$, and $ac = ce = 12{,}8$ feet; the vertical strain at c on each string $= \frac{W}{2} = \frac{5000}{2}$.

And, further, the actual strain in the direction of the string,

$$p. = q = \frac{W}{2} \cdot \frac{ac}{bc},$$

or

$$p = q = \frac{5000}{2} \times \frac{12{,}8}{8} = 4000 \text{ lbs.}$$

All other information is given in Fig. 8b.

When a heavy body, $ABCD$ (Fig. 8c), is suspended by two oblique strings, DH and CH, in a vertical plane, a straight line drawn through the intersection will pass through the centre of gravity, G, of the body.

8c.]

9.] For the force in the direction ad, represented by q, we find from Fig. 9,

$$gh = ab \cdot \frac{bd}{L} = ab \cdot \frac{W}{L};$$

$$W = gh + id = gh + q \cdot \cos \alpha;$$

$$W = ab \cdot \frac{W}{L} + q \cdot \frac{bd}{ad};$$

or
$$q = W \cdot \left(1 - \frac{ab}{L}\right) = W \cdot \frac{L - ab}{L} \cdot \frac{ad}{bd};$$

i. e.,
$$q = W \cdot \frac{bc}{L} \cdot \frac{ad}{bd};$$

and in the same manner from similarity of triangles,

$$p = W \cdot \frac{ab}{L} \cdot \frac{cd}{bd}.$$

In the equation for q is $W \cdot \frac{bc}{L}$, the vertical strain at d for the string ad; in the second equation is $W \cdot \frac{ab}{L}$, the vertical strain at d for the string cd,—equal to the shearing strains V and V_1 on the supports.

Example.—When, again,

$W = 5000$ lbs., $L = 100$ feet,

$ab = 10'$, $bc = 90'$, and $bd = 8'$,

$ad = 12{,}84'$, and $cd = 90{,}35'$,

so
$$p = 5000 \times \frac{10}{100} \times \frac{90{,}35}{8} = 5646 \text{ lbs.},$$

and
$$q = 5000 \times \frac{90}{100} \times \frac{12{,}84}{8} = 7020 \text{ lbs.};$$ 7222.5

then the results for the horizontal strain x and the vertical strain V at the right support are—

$$x = p \cdot \frac{cb}{cd} = p \cdot \sin \beta, \text{ or } x = W \cdot \frac{ab}{L} \cdot \frac{bc}{bd},$$

and
$$V = p \cdot \frac{bd}{cd} = p \cdot \cos . \beta, \text{ or } V = W \cdot \frac{ab}{L};$$

thus
$$x = 5000 \times \frac{10}{100} \times \frac{90}{8} = 5625 \text{ lbs.};$$

$$V = 5000 \times \frac{10}{100} = 500 \text{ lbs.}$$

The results for the horizontal strain x_1 and the vertical strain V_1 at the left support are—

$$x_1 = W.\frac{bc}{L}.\frac{ab}{bd},$$

and $$V_1 = W.\frac{bc}{L};$$

thus $$x_1 = 5000 \times \frac{90}{100} \times \frac{10}{8} = 5625 \text{ lbs.},$$

and $$V_1 = 5000.\frac{90}{100} = 4500 \text{ lbs.};$$

therefore, also, $$x = x_1,$$

for $$W.\frac{ab}{L}.\frac{bc}{bd} = W.\frac{bc}{L}.\frac{ab}{bd}. \qquad \text{(Fig. 17.)}$$

10.] For Fig. 10, when $W = 5000$ lbs.; ab, ad and bd the same as before—

$$Y = W.\frac{ad}{bd} = 5000 \times \frac{12,84}{8} = 8025 \text{ lbs.},$$

and $$Y_1 = W.\frac{ab}{bd} = 5000 \times \frac{10}{8} = 6250 \text{ lbs.}$$

11.] In Fig. 12,

$$P : Q : R = \sin.\, edb : \sin.\, adb : \sin\, adc.$$

12.] In general, for every triangle,

$$y = x + z, \qquad \text{(Fig. 12.)}$$

and, as here, $$x + y = z + m = R,$$

$$x + x + z = z + m,$$

or, $$m = 2x.$$

Also, from similarity of triangles ABC, ADC and DBC,

$$\frac{c}{b} = \frac{b}{a}, \qquad \text{or } c = \frac{b^2}{a};$$

and as $AB = c + a$, the diameter $= \frac{b^2}{a} + a$;

i. e., the diameter equals the square of one-half the chord divided by the height of the arc added to the height of the arc.

The height of the arc CB results from the chord CB in the same way.

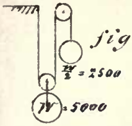


fig. 8 a
W/2 = 2500
W = 5000

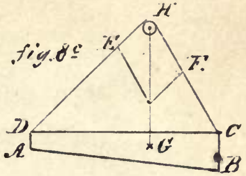


fig. 8 c
H
K
F
D
C
G
A
B

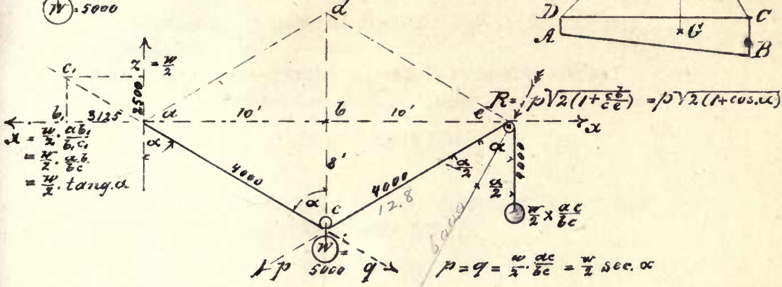


fig. 8 b
d
c1
z = W/2
2500
b1
3125
a
10'
b
10'
e
x
x = W/2 · ab1/b1c1
= W/2 · ab/bc
= W/2 · tang. α
R = p√2(1 + cb/ce) = p√2(1 + cos. α)
α
4000
8'
α
c
4000
12.8
α/2
α
α/2
4000
6400
W/2 × ac/bc
W
5000
p
q
p = q = W/2 · ac/bc = W/2 sec. α

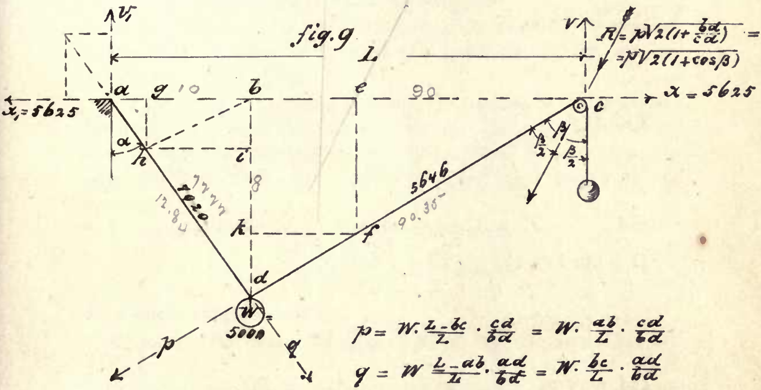


fig. 9
V1
L
V
R = p√2(1 + bd/cd) = p√2(1 + cos β)
x1 = 5625
a
g
10
b
e
90
c
x = 5625
α
h
i
8
7020
12.84
7777
k
f
5646
90.35
β
β/2
β/2
d
W
5000
p
q
p = W · (L − bc)/L · cd/bd = W · ab/L · cd/bd
q = W (L − ab)/L · ad/bd = W · bc/L · ad/bd

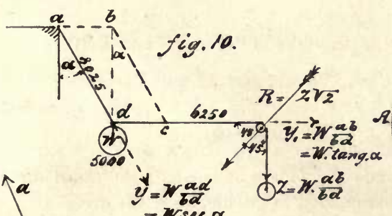


fig. 10.
a
b
α
α
8025
d
6250
c
R = Z√2
W
5000
45
45
y1 = W ab/bd
= W. tang. α
y = W ad/bd
= W sec. α
Z = W. ab/bd

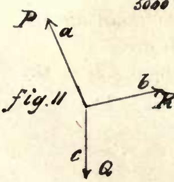


fig. 11
P
a
b
R
c
Q

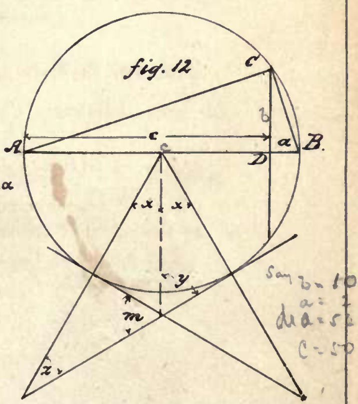


fig. 12
C
b
A
c
a
B
D
x
x
y
m
z
Say b = 10
a = 2
dia = 52
C = 50

Application in "Camber in Bridges," B, Sect. II. There, also, the geometrical rule,

$$\frac{\text{arc}}{\text{circumference}} = \frac{\text{angle at centre}}{360^\circ}.$$

[Plate 2—Figs. 8 to 12.]

III. *TRUSSES WITH SINGLE AND EQUALLY-DISTRIBUTED LOAD.*

Plate 3, Fig. 13.] A most frequent structure is the trussed beam (Fig. 13).

The post at the centre is called the king-post.

14.] The whole is a combined system, in which the horizontal beam, according to its stiffness, relieves the tie-rods from an aliquot amount of strain.

For the greatest exertion to which the tie-rods in the most unfavorable case could be exposed, we may use the result from Fig. 8^b, under the supposition that the horizontal beam counteracts only the horizontal forces. (For instance, when butted at the centre.)

To compute the stress we have the following:

$$p = q = \frac{W}{2} \cdot \frac{ac}{bc} = \frac{W}{2} \cdot \sec . \alpha,$$

for a single weight at centre.

Example.—The assumed weight = 20000 lbs.; between supports, 24 feet.

The length $bc = 5{,}59'$, and $ac = 13{,}24$ (which can be measured near enough for most purposes from a skeleton);

$$p = q = \frac{W}{2} \cdot \frac{ac}{bc} = 10{,}000 \times \frac{13{,}24}{5{,}59} = 23{,}700 \text{ (approx.)}.$$

The angle α being in this case 65°, for a calculation, using the preceding tables,

$$p = q = \frac{W}{2} \cdot \sec \alpha = \frac{20000}{2} \times \frac{1}{\cos \alpha} = 10000 \times \frac{1}{0{,}422} = 23700,$$

$$bc = ab \,.\, \text{tang } 25^\circ = 12 \times 0{,}4663 = 5{,}595,$$

$$ac = ab \,.\, \sec . 25^\circ = ab \,.\, \frac{1}{\cos . 25^\circ} = 12 \times \frac{1}{0{,}906} = 13{,}24.$$

The vertical pressure in the king-post under the supposition before $= 20{,}000$ lbs.; at each support $= 10{,}000$ lbs.; and the compression in the horizontal beam,

$$H = \frac{W}{2} \cdot \frac{ab}{bc} = 10000 \times \frac{12}{5{,}59} = 21467 \text{ lbs.}$$

15ª.] When in Fig. 15ª the strains p or q and the angle γ are known, we find the resulting vertical strain, R, also by means of the parallelogram of forces, viz.,

$$R = \sqrt{p^2 + q^2 + 2\,p \,.\, q \,.\, \cos \gamma};$$

or, because in this case $\alpha = \beta$, . therefore, $p = q$;

also, $\gamma = 2.65° = 130°$, or $= R + 40$,

and $\cos (R + 40) = -\sin 40$ (see preceding table),

so $R = \sqrt{2\,p^2 + 2\,p^2\,(-\sin 40)} = \sqrt{2\,p^2\,(1 - \sin 40)}$,

15ᵇ.] $R = \sqrt{2 \times 23700^2\,(1 - 0{,}642)} = 20000$ lbs.*

For a structure, Fig. 15ᵇ, reserved to the preceding one (15ª), the numerical value of strains is quite the same, but of opposite character, provided the enclosed angles are the same.

If, as in Fig. 16, the load $= 40000$ lbs., equally distributed on the beam, then each support will sustain again one-half of the load;

16.] but the reaction, D, of each support will be only one-quarter of the load $= 10000$ lbs.; and for the same exertion a truss or beam, charged with an equally-distributed load, will sustain twice as much as when loaded with a single weight at the centre. (Comp. Fig. 14.)

The distribution of forces on supports and at the centre is explained by Fig. 16.

For an angle, $\alpha = 63°\ 26'$, or, also, $\beta = 26°\ 34'$,

the depth of the truss being always equal to one-fourth of its length,

for $\text{tang} \,.\, 26°\ 34' = 0{,}5$.

Now, $\frac{bc}{ab} = \text{tang} \,.\, \beta$, or, $bc = db \,.\, 0{,}5 = 12 \times 0{,}5 = 6$,

---

* For the angle $aed = \delta$ in Fig. 15ᵇ,

$R = \sqrt{p^2 + q^2 + 2\,pq \cos \delta}$, and for the angle $edS = \varepsilon$,

$R = \sqrt{p_1{}^2 + p^2 - 2\,p_1 \,.\, p \,.\, \cos \varepsilon}$. (See Fig. 8ᵇ and Fig. 9.)

fig. 13.

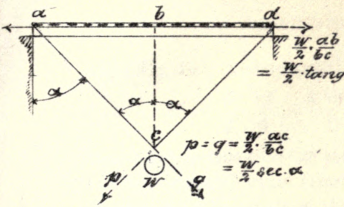


fig. 14.

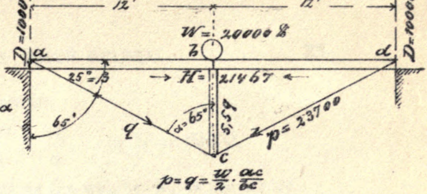


fig. 15 a

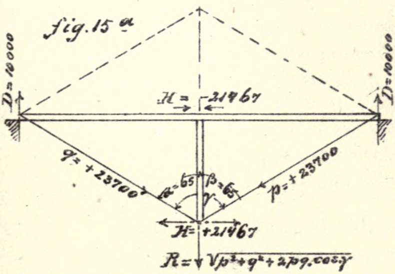


fig. 15 b

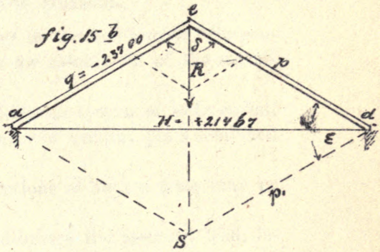
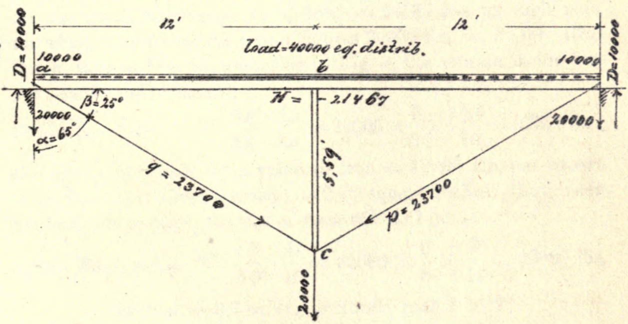


fig. 16.



and (Fig. 14), the horizontal strain,

$$H = \frac{W}{2} \cdot \frac{12}{6} = \frac{20000}{2} \times \frac{12}{6}, \quad \text{or } H = 20000,$$

thus being in this case the same as the weight, W, at the centre.

The horizontal strain (thrust) and the strain on the oblique rods increase with the angle α, thus being ∞ for an angle $\alpha = 90°$.

Suspension Truss Bridge.

We find a combination of trussing in the well-known "Suspension Truss" bridges, the principles for calculation of the strains being contained in the preceding.

The *Bollmann truss* forms a continuous system of independent trusses, in number equal to the number of vertical posts combined to a common top chord (stretcher).

Plate 4, Fig. 17. By Fig. 17 the dimensions of such a truss may be represented.

When for a single-track railroad-bridge the assumed load, including the weight of structure = $1\frac{1}{2}$ tons = 3360 lbs. per lineal foot, or for one rib (single truss) = 1680 lbs. per lineal foot—*i. e.*, for the given dimensions $12 \times 1680 = 20160$ lbs. on each post, for which may be said in round figures 20000 lbs. = W, for calculation, then in Fig. 17, according to Fig. 9, the tension in the first rod nearest the abutment,

$$\text{Strain No. 1, rod} = W \cdot \frac{bF}{AF} \cdot \frac{Ak}{bk} = 20000 \times \frac{7}{8} \times \frac{15{,}6}{10} = 27300 \text{ lbs.},$$

the section of which for a value of iron = 10000 lbs. per square inch (five to six times security) = 2,73 square inches; thus, when two rods are applied, the size of each rod = $1 \times 1\frac{3}{8}''$.

$$\text{Strain No. 2, rod} = W \cdot \frac{cF}{AF} \cdot \frac{Al}{cl} = 20000 \times \frac{6}{8} \times \frac{26}{10} = 39000 \text{ lbs.}$$

Section = 3,9 sq. in., or 2 rods, each $1 \times 2''$.

$$\text{Strain No. 3, rod} = W \cdot \frac{dF}{AF} \cdot \frac{Am}{dm} = 20000 \times \frac{5}{8} \times \frac{37{,}3}{10} = 46625 \text{ lbs.}$$

Sect. = 4,66, or 2 rods, each $1 \times 2\frac{3}{8}''$.

$$\text{Strain No. 4, rod} = W \cdot \frac{EF}{AF} \cdot \frac{AN}{EN} = 20000 \times \frac{4}{8} \times \frac{49}{10} = 49000 \text{ lbs.}$$

Sect. = 4,90, or 2 rods, each $1 \times 2\frac{1}{2}''$.

Strain No. 5, rod $= W \cdot \frac{fF}{AF} \cdot \frac{Ao}{fo} = 20000 \times \frac{3}{8} \times \frac{60,8}{10} = 45600$ lbs.

Sect. $= 4,56$, or 2 rods, each $1 \times 2\frac{1}{4}''$.

Strain No. 6, rod $= W \cdot \frac{gF}{AF} \cdot \frac{Ap}{gp} = 20000 \times \frac{2}{8} \times \frac{72,6}{10} = 36300$ lbs.

Sect. $= 3,63$, or 2 rods, each $1 \times 1\frac{7}{8}''$.

Strain No. 7, rod $= W \cdot \frac{hF}{AF} \cdot \frac{Aq}{hq} = 20000 \times \frac{1}{8} \times \frac{84,6}{10} = 21150$ lbs.

Sect. $= 2,11$, or 2 rods, each $\frac{7}{8} \times 1\frac{1}{4}''$.

When for a partial load at a certain panel the exertion of a pair of suspenders is greater than for a distributed load in calculation, those rods would be strained more than to one-fifth or one-sixth of their ultimate strength. So, when a locomotive of 84000 lbs. weight rests at a certain panel on a wheel-base of 12 feet, to each of the four supporting posts would be transmitted one-fourth of its weight = 21000 lbs.—this differing very little from the calculation in the example. Additional rods (panel-rods) are applied, sustaining the main suspenders and at the same time the top chord, transmitting and distributing the weight on the posts, these being always in a state of compression equal to the weight on the post.

Without the panel-rods for an over-grade bridge (through bridge) there would be in the post no further compression than that produced by the weight of the top chord and appendages, leaving for a strong cambered truss (B, Sect. II.), in case of a partial load, the possibility of raising.

The following is the strain in panel-rods according to Fig. 8^b:

$$\text{Strain} = \frac{W}{2} \cdot \frac{Er}{fr} = \frac{20000}{2} \times \frac{16}{10,5} = 15238 \text{ lbs.}$$

Sect. $= 1,52$, or 1 rod $= 1 \times 1\frac{1}{2}''$.

The influence of temperature upon the single systems of main suspenders (their length being different) is regulated by a link connection.

For the compressive strain in the top chord the rule for a girder, sustained at both ends and charged with an equally-distributed load, may be applied (see at the close of this chapter), then,

$$\frac{20000 \times \frac{7}{2} \times \frac{96}{4}}{10} = 168000.*$$

The compression in the top chord is the same all over.

For the result we have as momentum one-half of the entire weight on posts at one-fourth the length of truss as leverage,

or $$\text{Mom.} = \frac{Q}{2} \times \frac{L}{4},$$

which, when divided by the depth of truss, gives the compression = 168000 lbs. as before.

For a single load, P, at the centre would be

$$\text{Mom.} = \frac{P}{2} \times \frac{L}{2},$$

which, when divided by the depth of truss, gives for the compression twice as much, or 336000 lbs.; but for an addition of the results of each single truss with its single load, according to x and x_1 in Fig. 9, it would be

$$\text{Mom.} = \frac{P}{2} \times \frac{3\,L}{8},$$

this being one-half of the result for a single load, P, at the centre, added to one-half of the result for an equally-distributed load, Q.

18.] The *Fink truss* (Fig. 18) is different in principle.

Whilst in the Bollmann system there are as many independent trusses as there are posts, in the Fink all the trusses are dependent on each other and transfer the load toward the centre.

The centre post (king-post) has to sustain the compression of one-half of the entire load on the truss, including one-fourth of the weight of the rib, the main suspenders (tie-rods) depending again, as before, on the depth of the truss.

---

* For a simple compressive strain an area of section of stretcher = 9 square inches would be sufficient (18000 lbs.—safe load for cast iron)—the actual dimensions to be taken by Hodgkinson's formula on the strength of hollow cast-iron pillars:

$$W = \text{breaking weight in tons} = 44{,}3 \times \frac{D^{3.6} - d^{3.6}}{l^{1.7}};$$

therefore, when for a pillar, the external diameter, D, in inches, and the length, l, in feet, are known; and for six times security, with the weight, W, multiplied by 6, we can define the internal diameter, d, consequently the thickness of metal.

The calculation of an example in its simplicity will give the best explanation.

Taking the same dimensions and the same load as in the calculation for the preceding (Fig. 17), according to Fig. 8[b] we have,

$$\text{Strain in } A\,.\,N \text{ or } IN = \frac{W}{2} \times \frac{AN}{EN};$$

i.e.,
$$\frac{80000}{2} \times \frac{49}{10} = 196000 \text{ lbs.},$$

the section of which for a value of iron = 10000 lbs. per square inch = 19,6 square inches. Thus, when two rods are applied, the size of each = 2 × 5 inches.

$$\text{Strain in } kc,\ Ak \text{ or } cm = \frac{20000}{2} \times \frac{13}{5} = 26000 \text{ lbs.}$$

Section of a single rod = 1 × 2½ in. (full).

$$\text{Strain in } Al \text{ or } lE = \frac{40000}{2} \cdot \frac{26}{10} = 52000 \text{ lbs.}$$

Section of a single rod = 2 × 2½ in., or 1 × 5 in.

For a single locomotive (weight 84000 lbs.), resting at *cd* on a wheel-base of 12 feet, the vertical force for one post at *c* or *d* =

$$\frac{42000}{2} = 21000 \text{ lbs.} = W;$$

then,
$$\text{Strain in tie-rods} = \frac{21000}{2} \times \frac{13}{5} = 27300 \text{ lbs.};$$

so the size of rod *kc*, *Ak* or *cm* should be corrected to 1 × 2¾ in.

For the compressive strain on the top chord (stretcher), according to Fig. 14,

$$H = \frac{W}{2} \cdot \frac{AE}{EN},$$

and for a vertical strain $W = 80000$ lbs. in the centre post, as before mentioned,

$$H = \frac{80000}{2} \times \frac{48}{10} = 192000 \text{ lbs. (compr.).}$$

In this truss, as in the Bollmann, the compression in the top chord is the same all over.

For this truss, when applied for an over-grade railroad-bridge, a safe longitudinal connection (bracing or tying) will be essential on account of the variable load.

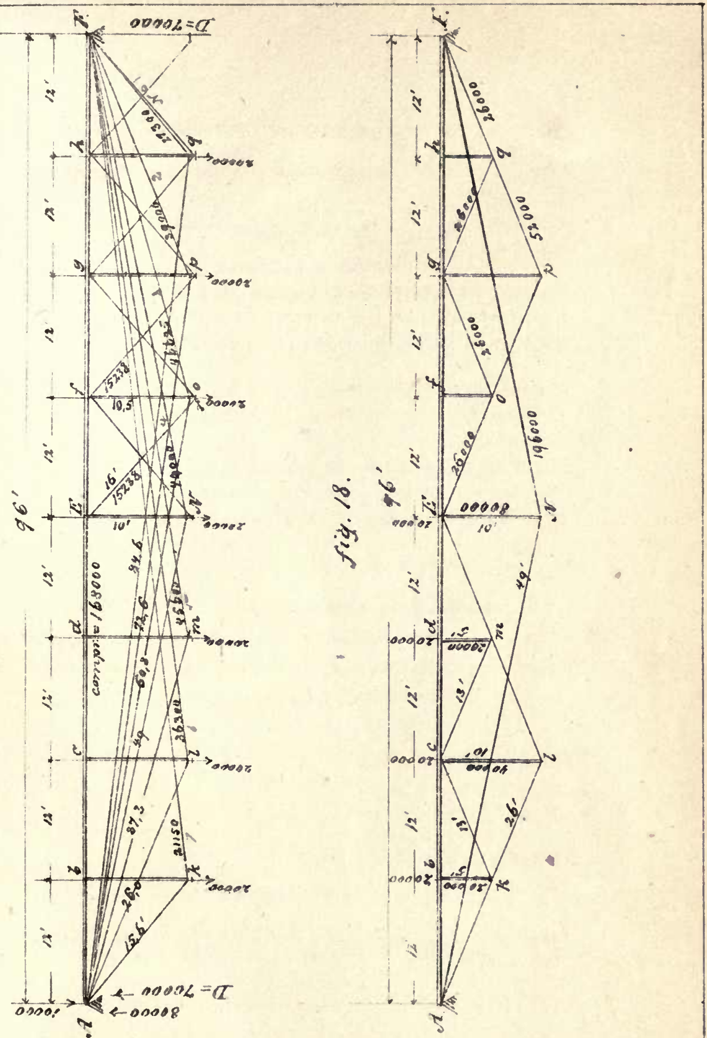


fig. 17

fig. 18.

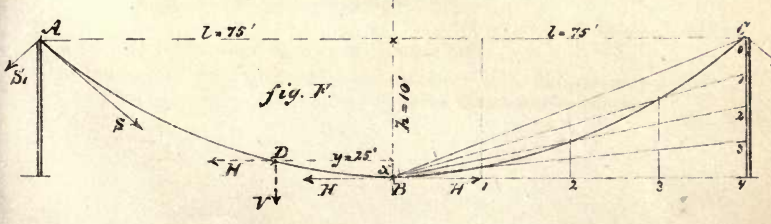


fig. F.

To the same category of bridges belongs the

SUSPENSION BRIDGE,

though, in regard to mathematics, very different.

The curve formed by a chain or cable lies between the parabola and the catenary, and is very nearly an ellipse. The curve in a loaded state approaches the parabola; in an unloaded state, the catenary. (Weisbach, vol. II.)

In the following example the curve may be considered a parabola or the bridge in its loaded state.

The thesis is—

The vertical force at every point of the chain equals the weight on the chain from the point in consideration unto the vertex.

Plate 4, Fig. F. So, when y in Fig. F = 25 feet and the length of bridge = 150 feet,

Width = 4 feet;

load, 50 lbs. per square foot, or 200 lbs. per lineal foot;

maximum load = 200 × 150 = 30000 lbs.;

the vertical force at D = 200 × 25 = 5000 lbs.

Further, *the horizontal force at every point of the chain is equal, and therefore equal to the horizontal strain in the vertex.*

Thus, when by p is represented the weight *pro unit* of horizontal projection = 200 lbs., for our example,

I. $H = \frac{pl^2}{2h}$, which is at the same time the horizontal force in A and C, to overturn the towers, and amounting here to $\frac{200 \times 75^2}{20} = 56250$ lbs.

II. $S = \frac{pl}{2h} \sqrt{l^2 + 4h^2} = \frac{15000}{20} \times 77{,}6 = 58500$ lbs.

III. $y^2 = \frac{l^2}{h} \cdot x.$

The length, L, of chain results by the formula,

$$L = 2l + \frac{4}{3} \times \frac{h^2}{l} = 151{,}75.*$$

* For more specifications I ought to refer to Weisbach and other authors.

For a trussed system with two posts (queen-posts), between supports 36 feet, and an assumed load $= 30000$ lbs., equally distributed, the distribution of forces on the bearings is denoted in Fig. 19, and for the calculation of strain, when compared with Fig. 10, we find for

Plate 5, Fig. 19.

$$Y = W.\frac{ad}{bd};$$

$$Y = 10000 \times \frac{13,24}{5,59} = 23700 \text{ (approx.)};$$

and for

$$Y_1 = W.\frac{ab}{bd};$$

$$Y_1 = 10000 \times \frac{12}{5,59} = 21467.$$

The compression on the vertical posts $= 10000$ lbs.; the vertical pressure on supports $= 15000$ lbs.; and, reduced by those 5000 lbs. directly sustained (comp. Fig. 16), the reactive force of supports, signified by $D = 10000$ lbs.

Upon this structure, the *Method of Moments* being applied (see Preface), we suppose a section separated from the original by a cut, *st*.

Considering the forces acting upon such a section, we form the equation of equilibrium for a suitable point of rotation, by solution of which we find the strain in the member in question; and observe the rule, that, *for a strain, Y, the point of rotation ought to be chosen in the intersection of x and z, making their lever $= 0$, when these are the members of the structure, separated by a cut, st, likewise as Y.*

20.] But when by *st* only one member, excepting Y, is separated, we lay the point of rotation on the next joint, as, per example, for Y in b, Fig. 20.

$$Y.5,07 = 10000 \times 12,$$

or, I., $\quad 0 = -Y.5,07 + 10000 \times 12$ (rotation round b);

$$Y = \frac{10000 \times 12}{5,07} = +23700;$$

and because in the following the form of equation always will be kept similar to I., the forces in their aim to turn to the left, like Y round b, will be signified by —, the same as a compressive strain; and the forces to the right, like the hands on a watch, or D round b, will be signified by +, the same as a tensile strain.

21.] According to this we have for Z, Fig. 21,

$$0 = Z \,.\, 5{,}59 + D \,.\, 12 \text{ (rot. } r \,.\, d),$$

$$Z = \frac{10000 \times 12}{5{,}59} = 21467,$$

and for Y_1, Fig. 22,

22.] $$0 = -Y_1 \,.\, 5{,}59 + D \,.\, 12 \text{ (rot. } r \,.\, b);$$

$$Y_1 = \frac{10000 \times 12}{5{,}59} = 21467.$$

When a diagonal, s, sustains the parallelogram, so that by a cut, st, three members, Z_1, Y_1 and s are separated, we have for the definition of s the point of rotation, as before mentioned, in the intersection of Z_1 and Y_1; but Z_1 and Y_1 in their direction are parallel, and therefore without intersection at all. In this case (the same as for diagonals in girders with parallel top and bottom flanges) we

23.] suppose a point of rotation, O, at any distance in the direction (axis) x, and find thus from Fig. 23, where $x = \infty$, or infinite—*i. e.*, the lever of all forces, acting in a vertical direction upon the section $= \infty$.

$$0 = -s \,.\, \infty \sin \varphi - D \,.\, \infty + p \,.\, \infty \quad \text{(rot. } r \,.\, O);$$

or, because ∞ is a factor of each part,

$$0 = -s \,.\, \sin \varphi - 10000 + 10000.$$

In this equation, $s \,.\, \sin \,.\, \varphi$ (comp. Fig. 56 on semi-girders) is

24.] the vertical component of s (Fig. 24), and acts right-angled to the axis, x, like D and p.

The angle φ for our example $= 25°$;

$$\sin \varphi = 0{,}422 \text{ (see table)},$$

and therefore $0 = -s \times 0{,}422 - 10000 + 10000,$

or $$s = 0,$$

showing that, as in Fig. 19, the diagonal, s, is without any strain and only useful for preventing dislocation. (Compare parabolic

25.] girders, to which this case is similar, because a parabola can be constructed through the sustaining points a, d, e and c, and therefore differs in this from Figs. 25 and 63.)

26.] For a reversed structure (Fig. 26) the strains will be the same, but of reversed signs.

In the following calculations of roofs and bridges it will be shown

that the *Method of Moments* is thoroughly applicable, leading directly and in the most comprehensive manner to distinct results; but for a preliminary estimate of strain in the top and bottom chords at the centre of the structure the most simple way to define this strain may be stated by the following:

As the flanged girder in Fig. 27, charged with an equally-distributed load, Q, will be exposed at its centre to the same
27, 28.] exertion as the girder in Fig. 28, fixed at the centre, the moments will be for both, when $V =$ depth of girder and $l =$ length (the load being equally distributed).

$$\text{Mom} = \frac{Q}{2} \cdot \frac{l}{4} = V.x \text{ (rot. } r.o);$$

$$\frac{\frac{Q}{2} \cdot \frac{l}{4}}{V} = \frac{\frac{1}{8} Q.l}{V} = \text{compression or tension in flanges;}$$

so $\qquad Q = 30000$ lbs.; $V = 5{,}59'$; and $l = 36'$.

$$\text{Mom} = \frac{30000}{2} \times \frac{36}{4} = 135000 = V.x,$$

and $\qquad \frac{135000}{5{,}59} = 24150$ lbs., the strain in the chords.

It is to be observed that the result is rather too high when applied upon a truss with few panels, as in Fig. 26, on account of the reactive pressure of the support, diminished by the partition of the direct load on this place.

On account of its being a very convenient method we recommend it; and in the following bridge skeletons we refer to it very frequently. (See D, Example I., and Sect II., note.)

29.] For a girder charged with a single weight at the centre (Fig. 29), we make a comparison with Fig. 30, and find for both

$$\text{Mom} = \frac{P}{2} \cdot \frac{l}{2} = V.x = \tfrac{1}{4} P.l,$$

30.] and $\qquad \frac{\frac{1}{4} P.l}{V} = \text{horizontal strain;}$

i. e., compression or tension in the top or bottom chords, the formation of moments for a point of rotation, o, being very comprehensive.

When $\qquad P = 15000$ lbs.,

fig. 19

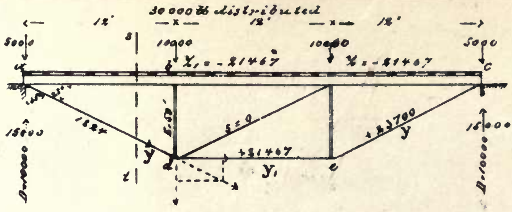


fig. 20

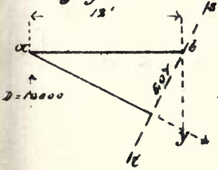


fig. 21

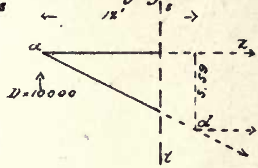


fig. 22

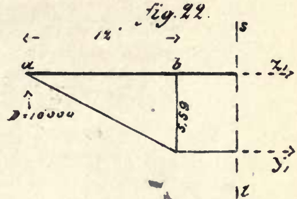
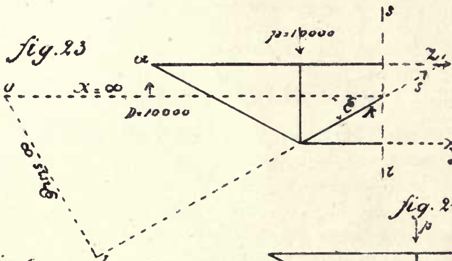


fig. 23



$\frac{ol}{ok} = \sin\varepsilon$

$ol = ok \sin\varepsilon = \infty \sin\varepsilon$

fig. 24

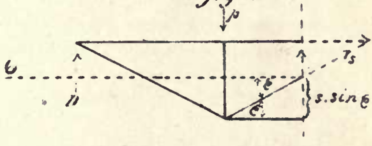


fig. 25

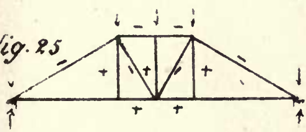


fig. 26

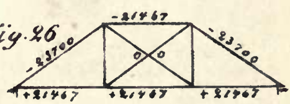


fig. 27.

fig. 28

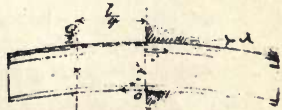


fig. 29

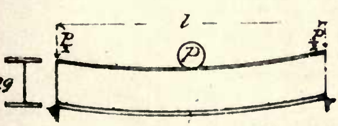


fig. 30.

but $V = 5,59$, and $l = 36$ (as before),

so the $$\text{Mom} = \frac{15000}{2} \times \frac{36}{2} = 135000,$$

and $$\frac{\frac{15000}{2} \times \frac{36}{2}}{5,59} = 24150 \text{ lbs., the strain in flanges;}$$

showing that, for the same exertion, a beam loaded with a single weight, P, at the centre can bear only one-half of an equally-distributed load, Q.

[Plates 3, 4 and 5—embracing Figs. 13 to 30.]

B. ROOF CONSTRUCTION.

For small and not complicated roofs, experience is a common and in general, also, a sufficient guide. But experience is very limited, and not every constructor has opportunity and time to acquire it.

The true and acceptable guide for a safe practice will always be the calculation; and since, especially for complicated and more extensive combinations, the application of mechanical science has become unavoidable, the following compendium, leading from the simplest to the most complicated structures, will give for almost every purpose sufficient information.

By construction, when EG represents the weight at the centre of gravity, G (Fig. 31ª), the body will be at rest when the plane DF is right-angled to the line DE; CE being horizontal—*i. e.*, right-angled to the vertical line EG. [Plate 6, Fig. 31ª.]

Let EG be an assumed length, then in the parallelogram of forces the intensity of EK and EH is measured in proportion by the same rule.

For an angle, $\beta = 25°$ of a rafter with the horizontal line (Fig. 31ᵇ—similar to Fig. 16 reversed) leaning with the top end against a wall, the heel at A being morticed; we then have [31ᵇ.]

$$\frac{cb}{ca} = \sin\beta = \frac{5,59}{13,24} = 0,422;$$

$$\sin^2\beta = 0,17;$$

$$\sin 2\beta = \sin 50° = 0,766;$$

$$\frac{ab}{ac} = \cos\beta = \frac{12}{13,24} = 0,906;$$

$$\cos^2\beta = 0,82.$$

When W is an equally-distributed load of 20000 lbs., then

$$H = H_1 = \frac{W}{2} \times \frac{d}{h} = \frac{20000}{2} \times \frac{12}{5,59} = 21467 \text{ lbs.}$$

The vertical force, V, at the top of the rafter $= 0$, and the vertical pressure V_2 of the heel $= 20000$ lbs., or equal to the entire load.

Also the vertical force, V_1 at the centre $= 20000$ lbs.

The pressure in the rafter itself (compression) $=$

$$\frac{W}{2} \times \frac{l}{h} = 10000 \times \frac{13,24}{5,59} = 23685 \text{ (23700 lbs.)}.$$

The entire pressure, R, of rafter toward the support,

$$R = \sqrt{V_2^2 + H_1^2} = 29300 \text{ lbs.}$$

Its direction can be constructed in making $K = 2h = 11,18$ feet, H_1 and V_2 forming the sides of the parallelogram.

When $P = 10000$ lbs., a single weight at centre of rafter,

$$H = H_1 = \frac{P}{2} \cdot \frac{d}{h} = 10733;$$

$$V = o,$$

and $$V_2 = V_1 = 10000 \text{ lbs.}$$

When in Fig. 31[c], by FM the weight of the body $ABCD$ is rep-

31[c].] resented, then FN, the force toward the wall, results in the horizontal and vertical forces CH and CV.

FL, the force acting perpendicular to the plane BI, in the direction BF.

G, the centre of gravity.

GF, vertical.

$CF \perp CB$, or $< FCB = 90°$.

$BI \perp FB$, then BI is the direction of the plane required to make the body at rest.

Also $$FN = P = \frac{s}{l} G \cos\alpha;$$

$$H = P \sin\alpha = \tfrac{1}{4} G \sin 2\alpha.$$

fig. 31a

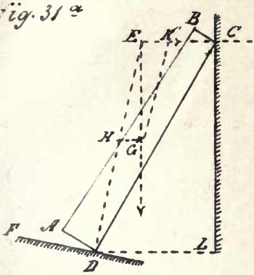


fig. 31b

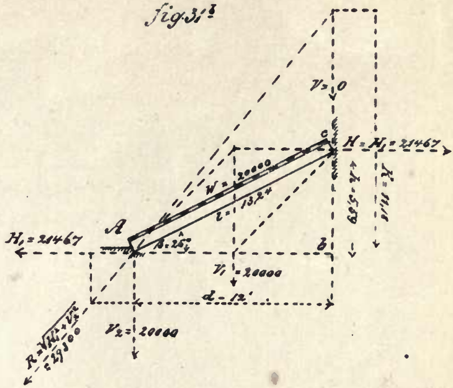


fig. 31c

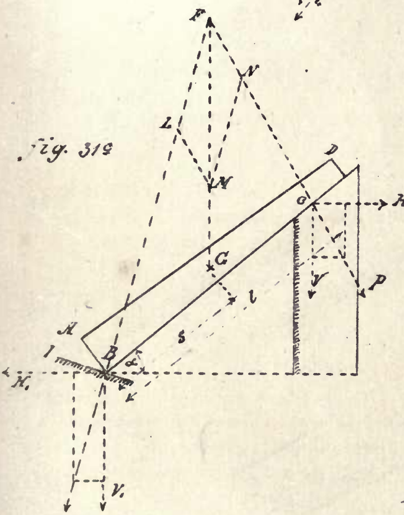


fig. 31e

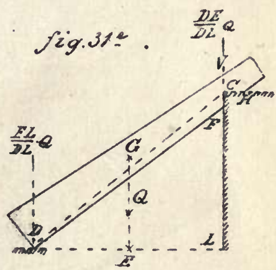
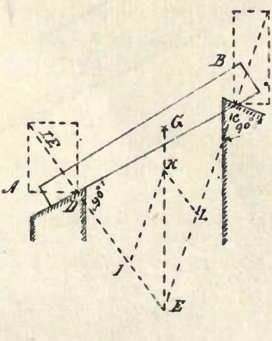


fig. 31d



31d.] Let the sloping body $ABCD$ (Fig. 31^d) be supported by a wall at its lower end, D, which coincides with the surface of the body;

Let G be again the centre of gravity;

It is required to cut a notch out of the body at the upper end, C, so that it may rest upon the top of a wall which is made to fit the notch.

Make GE vertical;

From D draw $DE \perp$ to CD;

Join EC, and draw CF at right angles to it; then the notch at C being cut, the body $ABCD$ will be at rest.

31e.] A body, $ABCD$, resting on supports (Fig. 31^e), will only produce the vertical strains $\frac{DE}{DL} \cdot Q$ and $\frac{EL}{DL} \cdot Q$ at the supports.

Plate 7, Fig. 32.] For Figs. 32, 33 and 34 we have, again,

$$\frac{bc}{ac} = \sin \beta; \ \frac{ab}{ac} = \cos \beta; \ \frac{bc}{ab} = \text{tang } \beta; \ \frac{ab}{bc} = \text{cotg } \beta;$$

$$\sin 2\beta = \sin 50^\circ = 0{,}766;$$

and when upon each rafter $W = 20000$ lbs., equally distributed, we have for Fig. 32,

$$H = H_1 = \frac{W}{2} \cdot \frac{d}{h} = \frac{W}{2} \cdot \text{cotg } \beta = 21467 \text{ lbs.}$$

The compression in the rafter $= \frac{W}{2} \cdot \frac{l}{h} = 23685$ lbs.,

and, again, $$R = \sqrt{H_1^2 + V_2^2} = 29300 \text{ lbs.}$$

For an angle $\beta = 26^\circ \ 34'$; the horizontal thrust will $= 20000$ lbs.—*i. e.*, the same as the entire weight. (Comp. Fig. 16.)

33.] For Fig. 33 is, as the rafter in a vertical direction, sustained on the top,

$$H = H_1 = \frac{W}{4} \cdot \sin 2\beta = 5000 \times 0{,}766 \times 3830;$$

$$V = \frac{W}{2} \cdot \cos^2 \beta = 10000 \times 0{,}82 = 8200;$$

$$V_2 = \frac{W}{2} (1 + \sin^2 \beta) = 10000 \ (1 + 0{,}17) + 11700;$$

and the compression in the rafter,

$$\frac{W}{2} \cdot \sin \beta = 10000 \times 0{,}422 = 4220 \text{ lbs.}$$

For Fig. 34, when the rafter is sustained at the top by a vertical post,

$$H = H_1 = \frac{W}{4} \cdot \sin 2\beta = 3830;$$

34.] $$V = W \cos^2 \beta = 20000 \times 0{,}82 = 16400;$$

$$V_2 = \frac{W}{2}(1 + \sin^2 \beta) = 10000\,(1 + 0{,}17) = 11700;$$

and the compression in the rafter =

$$\frac{W}{2} \cdot \sin \beta = 4220 \text{ lbs.}$$

In the cases in Figs. 33 and 34 the post relieves the tie-rod or (as here, in the absence of a tie) the wall from a part of the thrust of the rafters, and compared with the truss in the preceding we see that the king-post acts in a different way.

The different combinations of this single hanging-and-thrust construction may be disregarded, and by the following calculations the method of moments thoroughly applied.

A rafter being constructed in this way (Fig. 35), without connection between the point e and the horizontal tie-rod aa, there are two different systems, that of a trussed beam, aec, similar to Fig. 16,
35.] and that of a single triangle like Fig. 32, and it is as there for the same angle, β, and a load = 20000 lbs. upon each rafter; the horizontal thrust, H, at the top, as also the tension in the horizontal tie-rod = 21467 lbs. The trussed rafter is understood to be calculated like Fig. 16 in combination with Fig. 31[b].

When the weight of structure, snow and wind-pressure upon a
36.] roof (Fig. 36), for one square foot of horizontal projection, in all = 50 lbs., and the width between the supports or AC = 40 feet, the distance of rafters = 10 feet, then 20000 lbs. is the entire load, or 10000 lbs. upon each rafter, supported at A, B, C, D and E.

The pressure of one-half of the weight, or 10000 lbs., on the supports is counteracted by the direct load of 2500 lbs. It is therefore the reacting force, D, only 7500 lbs. (See Howe Truss, Sect. I., D.)

For a section separated by st we can define at once the strains in

fig. 32

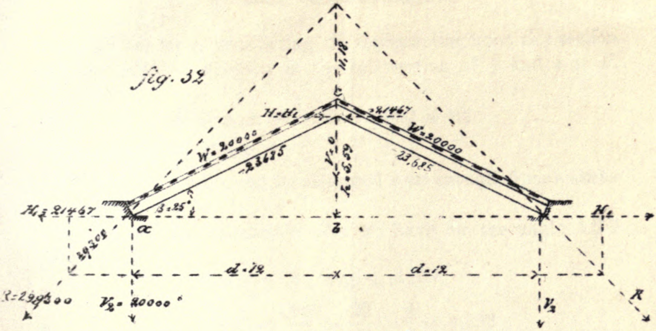


fig. 33

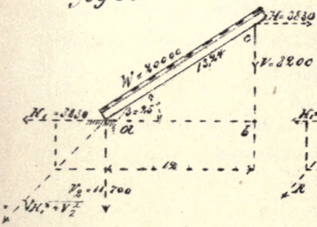


fig. 34

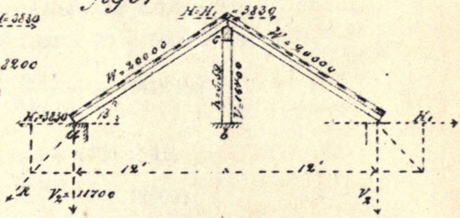


fig. 35

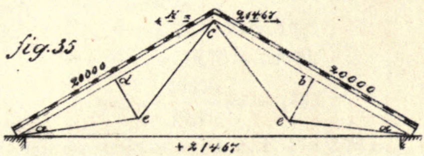


fig. 36.

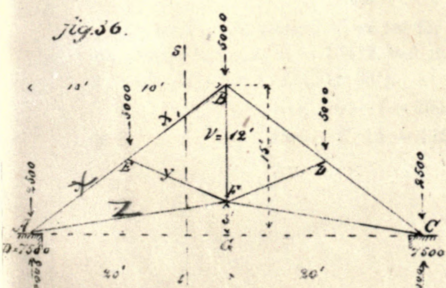
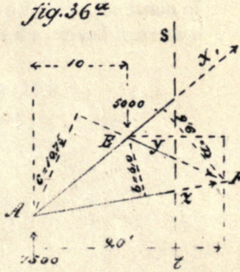


fig. 36^a



z, y and x_1, so for x_1, considering the forces acting upon this section and the point of rotation in the intersection of y and z or F. (Fig. 36[a].)

36[a].] $$0 = x_1 . a + D . 20 - 5000 \times 10.$$

(Comp. Fig. 20, Equat. I.)

The arm or lever, a, can be measured near enough from a skeleton $= 9{,}6'$.

Besides, for the calculation of a we have for the angle ABG (Fig. 36),

$$AG = GB . \text{tang} < ABG,$$

or $$\text{tang} . ABG = \frac{AG}{GB} = \frac{20}{15} = \frac{4}{3} = 1{,}333;$$

i. e., $$< ABG = 53^\circ\, 7', \text{ and } \sin 53^\circ\, 7' = 0{,}8;$$

$$a = BF \sin < ABG = 12 \times 0{,}8 = 9{,}6;$$

therefore $$0 = x_1 . 9{,}6 + 7500 \times 20 - 5000 \times 10;$$

$$x_1 = -\frac{100000}{9{,}6} = -10412 \text{ lbs.}$$

For z (rot. $r . E$, Fig. 36[a]),

$$0 = -z . 6 + 7500 \times 10,$$

or $$z = +\frac{75000}{6} = +12500;$$

and for y (rot. $r . A$, Fig. 36[a]),

$$0 = y . c + 5000 . 10;$$

$$0 = y . 10{,}75 + 50000,$$

or $$y = -\frac{50000}{10{,}75} = -4650.$$

Plate 8, Fig. 36[b].] For the strain in x (rot. $r . F$, Fig. 36[b]) is,

$$0 = x . 9{,}6 + 7500 \times 20,$$

or $$x = -\frac{150000}{9{,}6} = -15625.$$

In regard to the vertical V, we use for its definition the strain of the joining brace, $x_1 = -10412$, and make st a curved line; then we have, for a rot. $r . D$ (Fig. 36[b]),

$$0 = -V . 10 - (-10412) . 10{,}9,$$

or $$0 = -V . 10 + 113490;$$

i. e., $$V = + \frac{113490}{10} = + 11349.$$

37.] The results are combined in Fig. 37.

For Fig. 38, the entire load (equally distributed) again being
38.] 20000 lbs., the depth 15′, and between the supports 40′. When here the cut *st* separates the line $x_1y_1z_1$, we have for x_1 (rot. in the intersection of y_1z_1, or F, Fig. 39),

$$0 = x_1 . 9{,}1 - 5000 \times 5\tfrac{1}{2} + 7500 \times 15\tfrac{1}{2};$$

39.] $$x_1 = - \frac{88750}{9{,}1} = 9752.$$

For y_1 (rot. $r . A$), $0 = - y_1 . 15 + 5000 \times 10,$

or $$y_1 = + \frac{50000}{15} = + 3{,}333;$$

and for z_1 (rot. $r . C$),

$$0 = - z_1 . 15 - 5000 \times 10 + 7500 \times 20;$$

$$z_1 = + 6{,}666.$$

40.] For a section, *st*, through x and z, we have for x (Fig. 40),

$$0 = x . 9{,}1 + 7500 \times 15\tfrac{1}{2} \text{ (rot. } r . F);$$

$$x = - \frac{116250}{9{,}1} = - 12774;$$

and for z (rot. $r . D$),

$$0 = - z . 7{,}4 + 7500 \times 10;$$

or $$z = \frac{75000}{7{,}4} = 10135.$$

For y (rot. $r . A$) is, $0 = y . 12{,}5 + 5000 \times 10;$

$$y = - \frac{50000}{12{,}5} = - 4000.$$

41.] The results combined in Fig. 41.

42.] When the figure before is changed in the depth, like Fig. 42, we have the following equation:

$$0 = x_1 . 5{,}8 - 5000 \times 3{,}25 + 7500 \times 13{,}25 \text{ (rot. } r . F\text{, Fig. 43)};$$

$$x_1 = - \frac{83125}{5{,}8} = - 14332.$$

43.] For the definition of y_1, the intersection of x_1 and z_1 will be in G, and it is for G as rotation,

$$0 = - y_1 . 8{,}25 + 5000 \times 6 + 7500 \times 4; \quad \text{(Fig. 43.)}$$

fig. 36b.

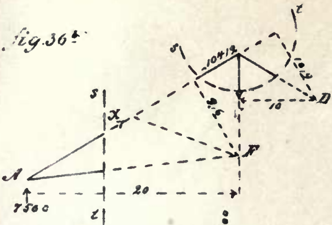


fig. 37.

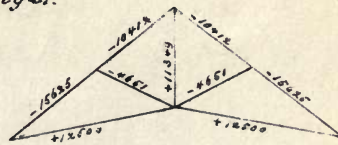


fig. 38

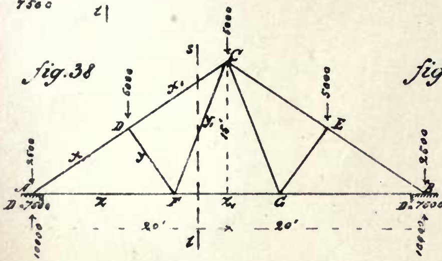


fig. 39

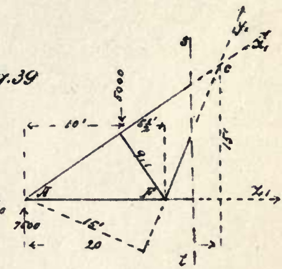


fig. 40

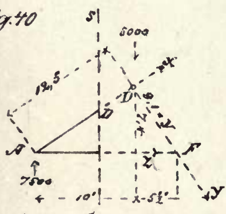


fig. 41

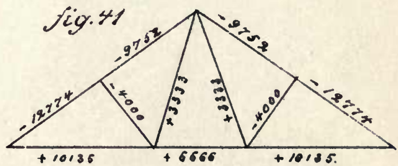


fig. 42.

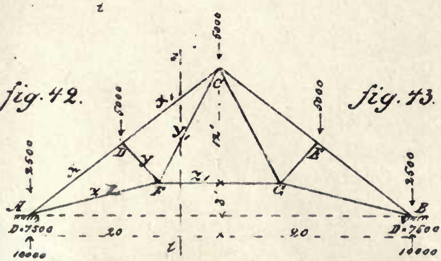


fig. 43.

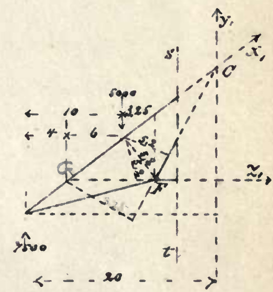


fig. 44.

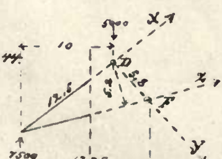
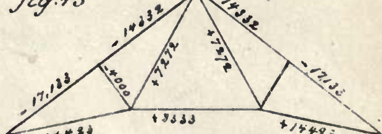


fig. 45



$$y_1 = + \frac{60000}{8,25} = 7272.$$

For z_1 (rot. $r \,.\, C$) we have

$$0 = - z_1 \,.\, 12 - 5000 \times 10 + 7500 \times 20;$$

$$z_1 = + \frac{100000}{12} = + 8333.$$

44.] For a section, *st*, through x and z (Fig. 44), it is

$$0 = x \,.\, 5,8 + 7500 \times 13,25 \text{ (rot. } r \,.\, F);$$

$$x = - \frac{993750}{5,8} = - 17133;$$

and $$0 = - z \,.\, 5,2 + 7500 \times 10 \text{ (rot. } r \,.\, D);$$

$$z = \frac{75000}{5,2} = + 14423;$$

and for y, $$0 = y \,.\, 12,5 + 5000 \times 10 \text{ (rot. } r \,.\, A);$$

$$y = - \frac{50000}{12,5} = - 4000.$$

45.] The results combined in Fig. 45.

Plate 9, Fig. 46.] For the definition of X in Fig. 46, the point of rotation in E, or in the intersection of Y and Z, will be from Fig 47.

47.] $$0 = X \,.\, x - P \,.\, CE + D \,.\, AE;$$

or $$X = \frac{P \,.\, CE - D \,.\, AE}{x}.$$

For Y we choose A, or the intersection of X and Z, as the point of rotation, and the equation will be

$$0 = - Y \,.\, y + P \,.\, AC + Q \,.\, AE,$$

or $$Y = \frac{P \,.\, AC + Q \,.\, AE}{y},$$

and in the same way for Z, rot. $r \,.\, H$.

$$0 = - Z \,.\, z - Q \,.\, EL - P \,.\, CL + D \,.\, AL,$$

or $$Z = \frac{- Q \,.\, EL - P \,.\, CL + D \,.\, AL}{z}.$$

It will not be necessary to show, by repetition of the foregoing, the equations for the other parts of the structure.

48.] In more complicated systems (Fig. 48), it may happen that by a cut, *st* (which can be made curved as well as straight), different braces or rods are spared, like FG, DG and DE.

In this case it is possible to come to a direct result when *st* only can be laid so that all the braces or rods cut by *st* meet at one point, except that one whose strain is in question.

49.] So for the strain V in FG (rot. $r.H$, Fig. 49),

$$0 = -V.FH - R.r;$$

$$V = -\frac{R.r}{FH}.$$

50.] In the same manner the strain U in DG (rot. $r.H$, Fig. 50),

$$0 = U.u - R.r;$$

$$U = \frac{R.r}{u};$$

thus we find also the strain in KT and LT.

51.] Being by the foregoing in possession of a value for U in DG, we find for the strain X in DF, Y in DE, and Z in CE the following equations from Fig. 51:

$$0 = X.DE + U.v - Q.NO - P.MO + W.AO \text{ (rot. } r.E);$$

$$0 = Y.AD + U.l + Q.AN + P.AM \text{ (rot. } r.A);$$

$$0 = -Z.z + W.AN - P.MN \text{ (rot. } r.D);$$

each one enabling us to obtain a direct result for the strain in question.

Plate 10, Fig. 52.] For a roof (Fig. 52), the weight of which, 11,3 lbs. per square foot of its horizontal plan, may be calculated 20 lbs. for wind pressure and snow, making together 31,3 lbs. per square foot.

The distance of rafters being 15⅓ feet, the width, 100 feet, makes for each rafter 15⅓ × 100 × 31,3 = 48000 lbs. (approx.).

The load at each apex, therefore, will be $\frac{48000}{8} = 6000$ lbs., the distribution of which is shown by the skeleton.

For the reactive force on the supports is again

$$D = 24000 - 3000, \qquad \text{or } D = 21000 \text{ lbs.}$$

There are, in all, seven times 6000 lbs. acting downward, and twice 21000 lbs. acting vertically upward upon the system.

fig.46

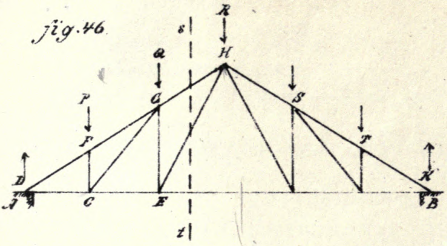


fig.47

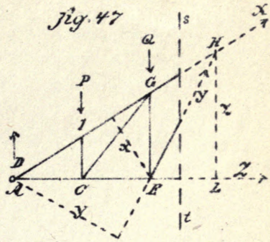


fig.48

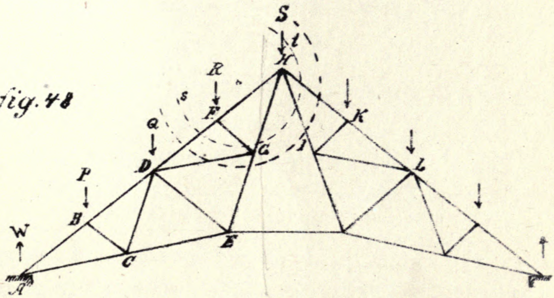


fig.49

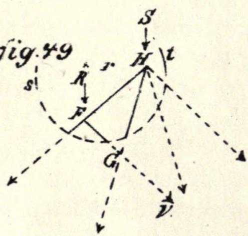


fig.50

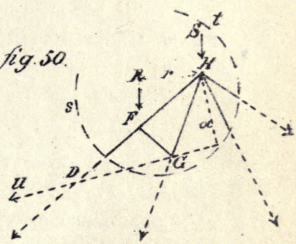
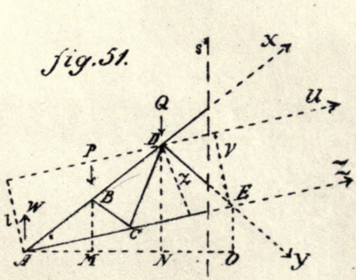


fig.51.



53.] The section, A, s, t (Fig. 53), kept in equilibrium by the replaced forces, x, y and z, may be regarded first as a lever with the fulcrum at D; then the strain in x for the middle section is

$$0 = x.18{,}6 + 21000 \times 50 - 6000 \times 12{,}5 - 6000 \times 25 - 6000 \times 37{,}5,$$

or

$$x = 32300 \text{ lbs.};$$

and in y, when A is the point of rotation,

$$0 = y.38{,}4 + 6000 \times 12{,}5 + 6000 \times 25 + 6000 \times 37{,}5 \text{ (rot. } A\text{)};$$

$$y = -11700 \text{ lbs.},$$

and

$$0 = -z.15 + 21000 \times 37{,}5 - 6000 \times 12{,}5 - 6000 \times 25 \text{ (rot. } r.E.\text{)},$$

$$Z = +37500 \text{ lbs.}$$

54.] For V in Fig. 54 the rotation also round A is

$$0 = -V.37{,}5 + 6000 \times 12{,}5 = 6000 \times 25;$$

$$V = +6000 \text{ lbs.}$$

For the other members in Fig. 52,

$$0 = x_1.13{,}9 + 21000 \times 37{,}5 - 6000 \times 12{,}5 - 6000 \times 25 \text{ (rot. } r.F\text{)};$$

$$x_1 = -40400;$$

$$0 = y_1.23{,}5 + 6000 \times 12{,}5 + 6000 \times 25 \text{ (rot. } r.A\text{)};$$

$$y_1 = -9570;$$

$$0 = -z_1.10 + 21000 \times 25 - 6000 \times 12{,}5 \text{ (rot. } r.G\text{)};$$

$$z_1 = +45000;$$

$$0 = -V_1.25 + 6000 \times 12{,}5 \text{ (rot. } r.A\text{)};$$

$$V_1 = +3000;$$

$$0 = x_2.9{,}3 + 21000 \times 25 - 6000 \times 12{,}5 \text{ (rot. } r.H\text{)};$$

$$x_2 = -48400;$$

$$0 = y_2.9{,}3 + 6000 \times 12{,}5 \text{ (rot. } r.A\text{)};$$

$$y_2 = -8100;$$

$$0 = -z_2.5 + 21000 \times 12{,}5 \text{ (rot. } r.I\text{)};$$

$$z_2 = +52500.$$

For the strain in x_3 we choose a convenient point for rotation in the line z, per Example D, Fig. 55.

55.] The equation in this case will be

$$0 = x_3 \,.\, 18{,}6 + 21000 \times 50;$$
$$x_3 = -56500.$$

The only strain not directly deducible is U in the vertical line CD at the centre.

As in Fig. 36, we use the strain of the joining brace,

$$x = -32300 \text{ lbs.}$$

56.] For B as the point of rotation (Fig. 56), our equation is

$$0 = -U \,.\, 50 - 6000 \times 50 - (-32300) \,.\, 37{,}2;$$
$$U = 18000 \text{ lbs.}$$

57.] The results combined in Fig. 57.

58.] The weight and load of a roof (Fig. 58) being estimated, including wind-pressure and snow, to 50 lbs. per square foot of its horizontal plan, the distance of rafters being 12 feet, and the space between the walls 99 feet, which gives $50 \,.\, 12 \,.\, 99 = 59400$ lbs. for one rafter, or, in round figures, 60000 lbs.

The calculation of the top structure can be made as in the preceding example (Fig. 36).

58.] In the main construction are six supporting points, charged as in Fig. 58. The top structure transmits one-third of the entire load, or on each post 10000 lbs. to the apexes, *ff*.

Each wall has to bear 30000 lbs.; and after subtraction of the direct load the reactive force is 26666 lbs., or, by calculation,

$$D = \frac{6666\,(11 + 22)}{99} + \frac{13333\,(33 + 66)}{99} + \frac{6666\,(77 + 88)}{99};$$
$$D = 26666 \text{ lbs.}$$

59.] For the strain x_3 we have in Fig. 59,

$$0 = -x_3 \,.\, 21 - 13333 \times 16\tfrac{1}{2} - 6666\,(27\tfrac{1}{2} + 38\tfrac{1}{2}) + D \,.\, 49\tfrac{1}{2} \quad (\text{rot. } r \,.\, h);$$

or, also,

$$0 = -x_3 \,.\, 21 - 6666\,(11 + 22) + 26666 \times 33 \ (\text{rot. } r \,.\, f);$$
$$x_3 = + \frac{659967}{21} = + 31427.$$

Further,

$$0 = Z_4 \,.\, 21 - 13333 \times 16\tfrac{1}{2} - 6666\,(27\tfrac{1}{2} + 38\tfrac{1}{2}) + D \,.\, 49\tfrac{1}{2} \quad (\text{rot. } r \,.\, g).$$

fig. 52

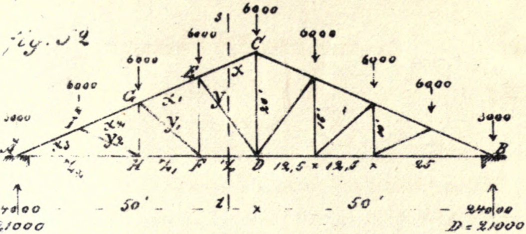


fig. 53

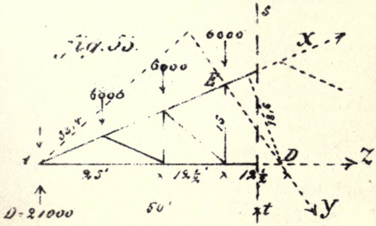


fig. 54

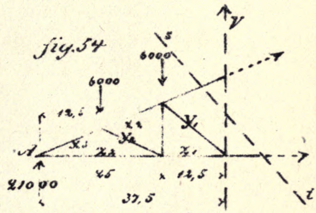


fig. 55

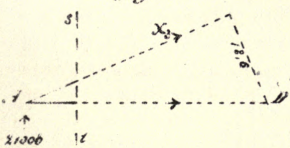


fig. 56

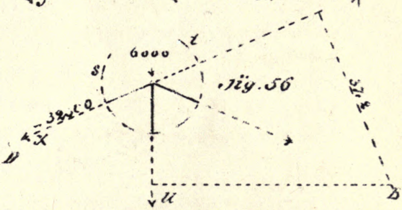


fig. 57

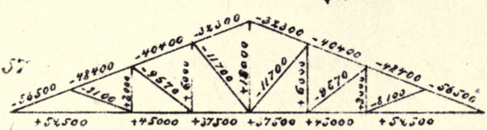
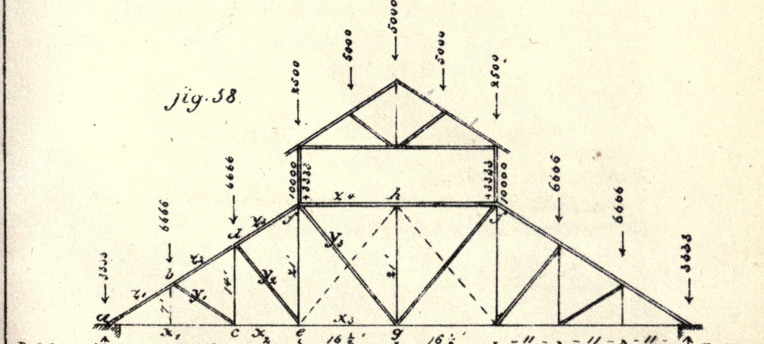


fig. 58



$$Z_4 = -\frac{659967}{21} = -31427,$$

and $0 = y_3 . 39{,}5 + 13333 \times 33 + 6666\,(11 + 22)$ (rot. $r . a$);

$$Y_3 = -16708.$$

The tie-rod, gh, transmits the strain to the top flange, and is here sustained by the counter-brace, eh.

60.] From Fig. 60 is

$$0 = -x_2 . 14 - 6666 \times 11 + D . 22 \text{ (rot. } r, d);$$

$$x_2 = \frac{513304}{14} = 36665;$$

$$0 = z_3 . 17\tfrac{3}{4} - 6666\,(11 + 22) + 26666 \times 33 \text{ (rot. } r . e);$$

$$z_3 = -\frac{660000}{17{,}75} = -37180;$$

$$0 = y_2 . 26{,}7 + 6666 \times 11 + 6666 \times 22 \text{ (rot. } r . a);$$

$$y_2 = -8223.$$

61.] Fig, 61 gives the equations,

$$0 = -x_1 . 7 + 26666 \times 11 \text{ (rot. } r, b);$$

$$x_1 = +41902;$$

$$0 = z_2 , 13 - 6666 \times 11 + 26666 \times 22 \text{ (rot. } r . c);$$

$$z_2 = -39485;$$

$$0 = y_1 , 13 + 6666 \times 11 \text{ (rot. } r, a);$$

$$y_1 = -5640;$$

and for z_1 we find from the same figure,

$$0 = z_1 . 13 + 26666 \times 22;$$

$$z_1 = -45125.$$

62.] For the strain in tie-rods we find from Fig. 62.

$$0 = -V_3 , 33 + 6666 . (11 + 22) \text{ (rot. } r, a);$$

$$V_3 = +6666; \qquad \text{(Comp. Fig. 68.)}$$

$$0 = -V_2 . 22 + 6666 \times 11 \text{ (rot. } r . a);$$

$$V_2 = +3333;$$

$$0 = -V_1 . 11 + 0 \text{ (rot. } r . a);$$

$V_1 = 0$ (and is therefore not essential).

63.] The strain in V_4 at the centre rod, according to 8^b, can be defined thus:

$$V_4 = 2 \times \frac{21}{26,7} \times 16708 = 26200 \text{ lbs.}$$

The results are combined in Fig. 63.

When in Fig. 64 the rafters are trussed—*i. e.*, stiffened by a
64.] king-post at b—there will be only four supporting points in the main construction, because the load in this case is transferred to the wall.

$$D = \frac{9999 \times 22}{99} + \frac{13333\,(33 + 66)}{99} + \frac{9999 \times 77}{99} = 23332 \text{ lbs.}$$

Plate 12, Fig. 65.] Further in Fig. 65,

$$0 = x_2 \,.\, 21 - 13333 \times 16\tfrac{1}{2} - 9999 \times 27\tfrac{1}{2} + 23332 \times 49\tfrac{1}{2} \quad (\text{rot. } r \,.\, h);$$

$$x_2 = 31427;$$

$$0 = z_3 \,.\, 21 - 13333 \times 16\tfrac{1}{2} - 9999 \times 27\tfrac{1}{2} + 23332 \times 49\tfrac{1}{2} \quad (\text{rot. } r \,.\, g);$$

$$z_3 = -31427;$$

$$0 = y_2 \,.\, 39,5 + 9999 \times 22 + 13333 \times 33 \ (\text{rot. } r \,.\, a);$$

$$y_2 = -16708.$$

66.] And from Fig. 66,

$$0 = -x_1 \,.\, 14 + D \,.\, 22 = -x_1 \,.\, 14 + 23332 \times 22 \ (\text{rot. } r \,.\, d);$$

$$x_1 = +36665;$$

$$0 = z_2 \,.\, 17\tfrac{3}{4} + 23332 \times 33 - 9999 \times 11 \ (\text{rot. } r \,.\, e);$$

$$z_2 = -37180;$$

$$0 = y_1 \,.\, 26,75 + 9999 \times 22 \ (\text{rot. } r \,.\, a);$$

$$y_1 = -8223.$$

67.] For Z we have from Fig. 67,

$$0 = Z_1 \,.\, 13 + 23332 \times 22 \ (\text{rot. } r \,.\, c);$$

$$Z_1 = -39485.$$

68.] See the results in Fig. 68 combined.

The strain $V_4 = 26200$ lbs. can be defined independently of the Method of Moments by the parallelogram of forces, as in Fig. 63, already shown,

$$V_4 = 2\,(-16708) \,.\, \cos \alpha;$$

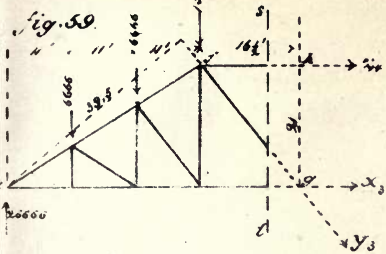


Fig. 59.

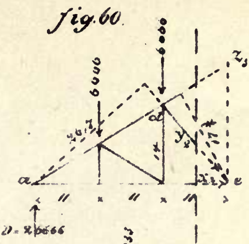


Fig. 60.

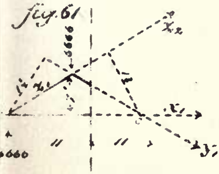


Fig. 61.

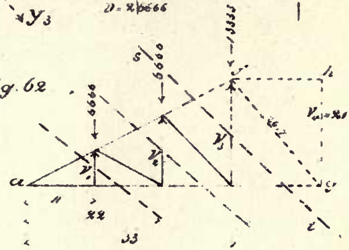


Fig. 62.

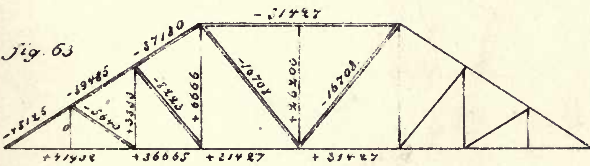


Fig. 63.

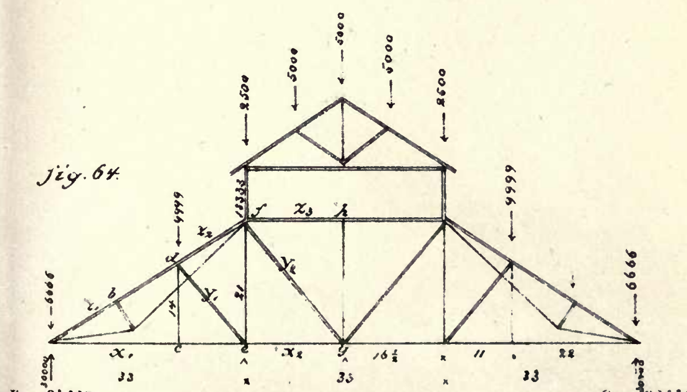


Fig. 64.

fig. 65

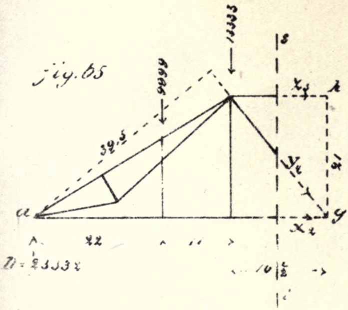


fig. 66

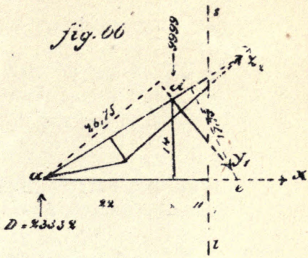


fig. 67

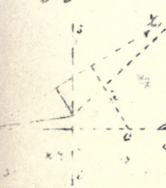


fig. 68

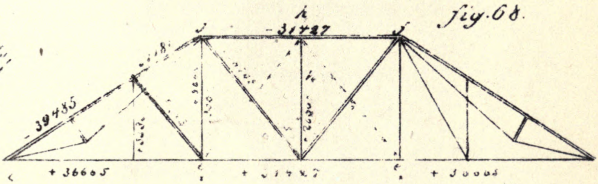


fig. 69

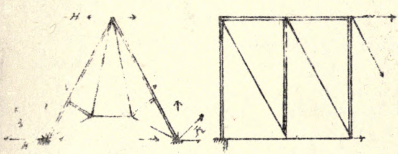
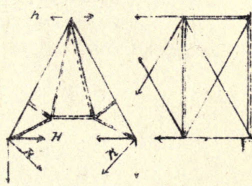


fig. 70



and when by means of counter-braces, e, h, the top chord is relieved from the strain, so that one-half to each side is transported to the tie-rods, e, f, then here the strain will increase to $13000 + 6666 = 19666$ lbs.

69, 70.] In a combination of rafters (Figs. 69, 70), the pressure of the end rafters upon the wall results in an outward horizontal and vertical force.

Different from this is the action of the intermediate rafters, being similar to an oblique bridge-truss, sustained at the top chord.

The horizontal force at the heels of the intermediate rafters is opposed to the horizontal force of the end rafters.

[Plates 6, 7, 8, 9, 10, 11 and 12—embracing Figs. 31 to 70.]

C. SEMI-GIRDERS.

I. *SEMI-GIRDERS LOADED AT THE EXTREMITY.*

Plate 13, Fig. 71.] As the most simple presentation for a weight, W, the stress in struts and tie-rods is inscribed in Figs. 71 to 74, and the parallelogram of forces connected.

73.] To compute in Fig. 73 the stress in the lower flange, we have

$$\frac{de}{df} = \sec \alpha, \qquad \text{and} \quad \frac{\frac{Z}{2}}{-W.\sec\alpha} = -\sin\alpha,$$

or

$$\frac{Z}{2} = -W.\sec\alpha.\sin\alpha;$$

$$Z = -W.\sec\alpha.\sin\alpha - W.\sec\alpha\sin\alpha,$$

or

$$Z = -2W.\sec\alpha.\sin\alpha;$$

and since

$$\sin\alpha = \frac{\text{tang}\,\alpha}{\sec\alpha},$$

$$Z = -2W.\sec\alpha\frac{\text{tang}\,\alpha}{\sec\alpha} = -2W.\text{tang}\,\alpha$$

(much easier determined in Fig. 77 by the Method of Moments).

From Fig. 73 and the following we see that, for a load at the extremity, the diagonals are strained equally and alternately with tensile (+) and compressive (—) strains. (Comp. Fig. 23.)

But the strain in the flanges increases toward the support in each,

$$2\,W.\,\text{tang}\,\alpha\,,$$

where α is the angle of diagonals with a vertical line.

75.] For a better presentation of this, see Fig. 75, and for the calculation apply the Method of Moments.

76, 77.] When by a cut, st, a section of the structure is separated, we have for the flanges as equation of equilibrium (Figs. 76 and 77),

$0 = -x_1\,.\,cb + W\,.\,ca$ (rot. $r\,.\,b$), or for $cb = h$, and $ac = l$;

$$x_1 = +W\cdot\frac{l}{h} = +W.\,\text{tang}\,\alpha\,;$$

$0 = +z_1\,.\,h + W.\,2\,l$ (rot. $r\,.\,d$);

$$z_1 = -2W\cdot\frac{l}{h} = -2W.\,\text{tang}\,\alpha\,;$$

78, 79] and by Figs. 76 and 79:

$0 = -x_2\,.\,h + W.\,3\,l$ (rot. $r\,.\,e$);

$$x_2 = +3W\cdot\frac{l}{h} = 3W.\,\text{tang}\,\alpha.$$

$0 = +z_2\,.\,h + W.\,4\,l$ (rot. $r\,.\,f$);

$$z_2 = -4W\cdot\frac{l}{h} = -4W.\,\text{tang}\,\alpha\,.$$

In the same manner is

$0 = -x_3\,.\,h + W.\,5\,l$ (rot. $r\,.\,g$);

$$x_3 = 5W\cdot\frac{l}{h}$$

$0 = +z_3\,.\,h + W.\,6\,l$ (rot. $r\,.\,h$);

$$z_3 = -6W\cdot\frac{l}{h}.$$

Cranes.

Plate 14, Fig. A.] A wrought-iron crane (Fig. A), constructed of braces with link-joints, may be loaded at the extremity with 30000 lbs. $= P$; so is (for the dimensions noted in the skeleton) the horizontal strain, S, in a and b.

$$0 = -S\,.\,6 + P\,.\,12 \text{ (rot. } r\,.\,a);$$

$$S = 60000 \text{ lbs};$$

B.] and for the other members we have

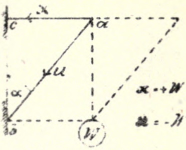
$0 = -z_1\,.\,0{,}75 + 0$ (rot. in the intersection of y_1 and x_1 or g, Fig. B);

$$z_1 = 0;$$

$$o = y_1\,.\,1{,}1 - P\,.\,2{,}1 \text{ (rot. } r\,.\,i);$$

$$y_1 = +\frac{30000 \times 2{,}1}{1{,}1} = 57272;$$

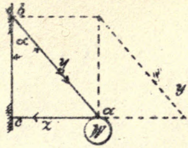
fig. 71.



$x = +W \cdot \frac{ca}{cb} = +W \cdot \text{tang.}\alpha$

$u = -W \cdot \frac{ba}{bc} = -W \cdot \sec.\alpha$

fig. 72

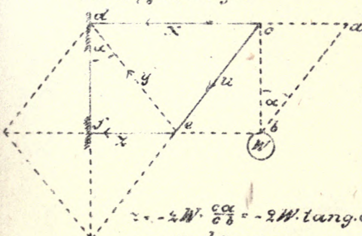


$y = +W \cdot \frac{ba}{bc} = +W \cdot \sec.\alpha$

$z = -W \cdot \frac{ca}{cb} = -W \cdot \text{tang.}\alpha$

fig. 73

$x = W \frac{ca}{cb} = W \cdot \text{tang.}\alpha$



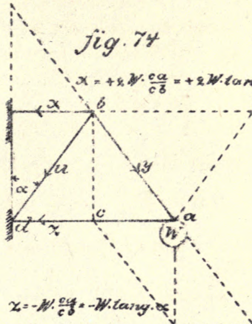
$z = -2W \cdot \frac{ca}{cb} = -2W \cdot \text{tang.}\alpha$

$y = +W \cdot \frac{ba}{bc} = +W \cdot \sec.\alpha$

$u = -W \cdot \frac{ba}{bc} = -W \cdot \sec.\alpha$

fig. 74

$x = +2W \cdot \frac{ca}{cb} = +2W \cdot \text{tang.}\alpha$

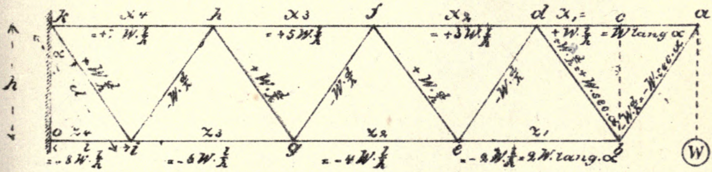


$z = -W \cdot \frac{ca}{cb} = -W \cdot \text{tang.}\alpha$

$y = +W \cdot \frac{ba}{bc} = +W \cdot \sec.\alpha$

$u = -W \cdot \frac{ba}{bc} = -W \cdot \sec.\alpha$

fig. 75



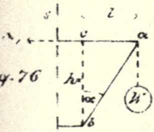
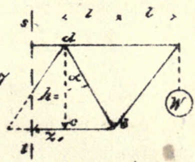


fig. 76

fig. 77



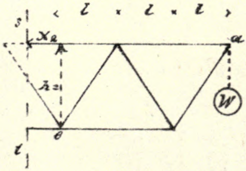
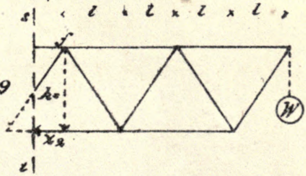


fig. 78

fig. 79



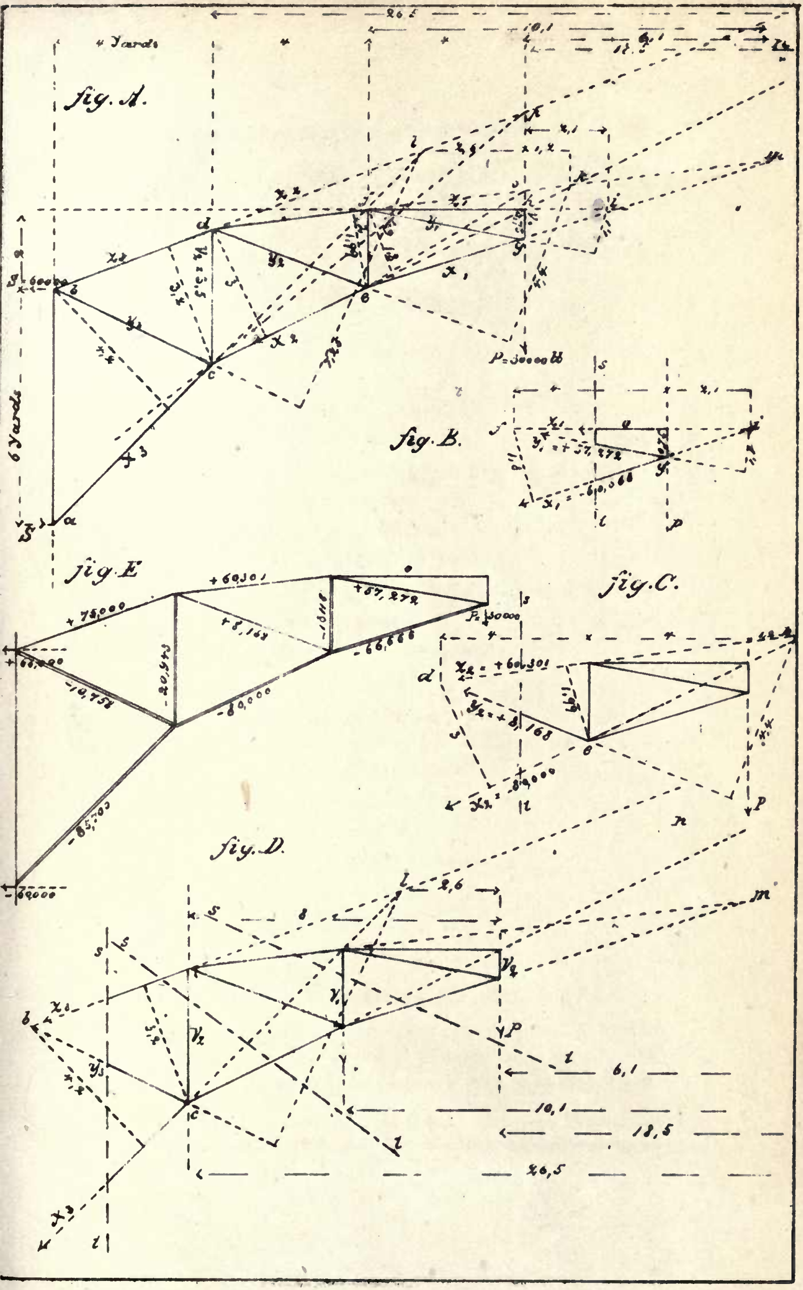


fig. A.
4 Yards
6 Yards
P=30000 lb
fig. B.
fig. C.
fig. D.
fig E
+75,000
+60,301
+57,272
+8,168
-66,666
-80,000
-85,700
-10,752
26,5
10,1
18,5
6,1

$$0 = x_1 . 1{,}8 + P . 4 \text{ (rot. } f);$$

$$x_1 = -\frac{30000 . 4}{1{,}8} = -66666;$$

C.] $$0 = -z_2 . 1{,}99 + P . 4 \text{ (rot. } r . e, \text{ Fig. C)};$$

$$z_2 = \frac{120000}{1{,}99} = +60301;$$

$$0 = y_2 . 4{,}4 - P . 1{,}2 \text{ (rot. } r . k);$$

$$y_2 = +8168;$$

$$0 = x_2 . 3 + P . 8 \text{ (rot. } r . d);$$

$$x_2 = -80000;$$

D.] $$0 = -z_3 . 3{,}2 + P . 8 \text{ (rot. } r . c, \text{ Fig. D)};$$

$$z_3 = 75000;$$

$$0 = x_3 . 4{,}2 + P . 12 \text{ (rot. } r . b);$$

$$x_3 = 85700.$$

For y_3 the intersection l of x_3 and z_3 is to the left of the suspended weight, and the symbol reversed.

$$0 = y_3 . 7{,}25 + P . 2{,}6 \text{ (rot. } r . l);$$

$$y_3 = -10758,$$

which would be $= 0$ when the intersection is in the vertical line of the suspended weight, as the lines oe and pc in Fig. A indicate.

For the verticals, V, we have from Fig. D,

$$0 = -V_1 . 10{,}1 - P . 6{,}1 \text{ (rot. } r . m);$$

$$V_1 = 18118;$$

$$0 = -V_2 . 26{,}5 - P . 18{,}5 \text{ (rot. } r . n);$$

$$V_2 = -20943.$$

E.] The results combined in Fig. E.*

II. *SEMI-GIRDERS LOADED AT EACH APEX.*

In Fig. 25 is occasionally explained how to compute the stress in diagonals, as there is no intersection of joining flanges, x and z, and as in the case here considered the diagonals receive at each loaded

---

* For most purposes the above will be sufficient. In Glynn's rudimentary treatise on the Construction of Cranes we find valuable and complete drawings

Plate 15, Fig. 80.] apex an increment of strain, prior to the calculation may be given the general thesis that *the strain in two diagonals whose intersection is at an unloaded point is the same in numerical value, but of opposite character.* (Fig. 80.)

(See IV. General Remarks.)

The strain in diagonals, meeting at a loaded point, is in numerical value different.

The strain in flanges increases from apex to apex in geometrical progression.

81.] In Fig. 81 suppose the angle φ of diagonals with a horizontal line $= 45°$, so, also, angle $\propto = 45°$; and by the table, $\sec \propto = 1{,}414$.

When, again, in the axis $x = \infty$ a point of rotation, o, is supposed, we have per example for diagonal, y_5.

$$0 = + y_5 . x \sin \varphi + \left(W + W + \frac{W}{2} \right) . x \text{ (rot. } r . o),$$

where $y_5 . \sin \varphi$ is the vertical component of y_5, or $= ab = a_1 b_1$ in the parallelogram of forces (Fig. 81), presenting by y_5 the resulting strain or diagonal. Divided by x, it follows:

$$0 = + y_5 \sin \varphi + \left(W + W + \frac{W}{2} \right);$$

and as $\varphi = 45°$, and $\sin 45° = 0{,}707$,

$$0 = + y_5 . 0{,}707 + \tfrac{5}{2} . W,$$

or

$$y_5 = - 3{,}535 \, W.$$

The same results from $- \tfrac{5}{2} W . \sec \propto$, or $- \tfrac{5}{2} . W . 1{,}414$, which is also $= - 3{,}535 \, W$.

82.] In the same manner in Fig. 82 for y_2, the direction of which, when separated by a cut, *st*, is reversed to the weight. (See Figs. 22 and 23.)

$$0 = - y_2 . \sin 45 + \frac{W}{2} \text{ (rot. } o);$$

$$y_2 = + \frac{\frac{1}{2} W}{0{,}707}, \quad \text{and } y_1 = - \frac{\frac{1}{2} W}{0{,}707};$$

and for the other diagonals,

$$0 = + y_3 . 0{,}707 + W + \frac{W}{2},$$

or

$$y_3 = - \frac{\frac{3}{2} W}{0{,}707}, \quad \text{and } y_4 = + \frac{\frac{3}{2} W}{0{,}707};$$

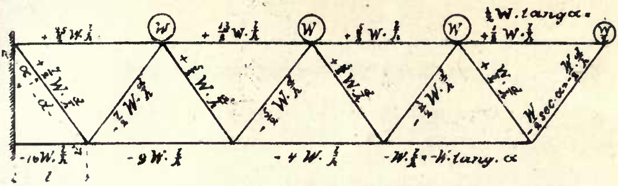


fig. 80

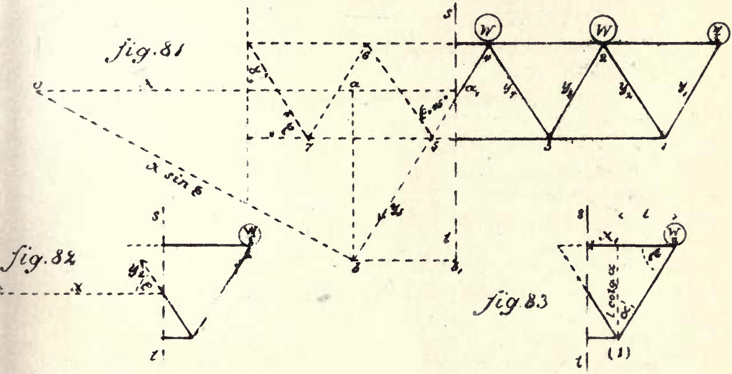


fig. 81

fig. 82

fig. 83

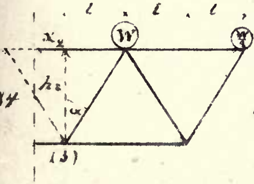


fig. 84

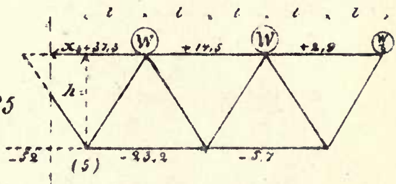


fig. 85

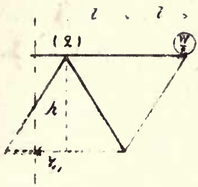


fig. 86

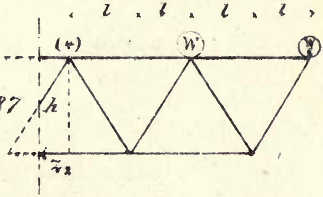


fig. 87

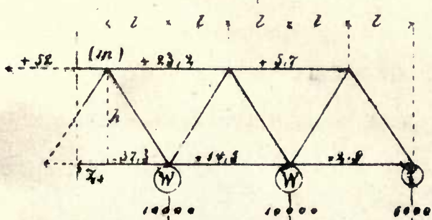


fig. 88

$$y_5 = -\frac{\frac{5}{2}W}{0{,}707}, \qquad \text{and } y_6 = +\frac{\frac{5}{2}W}{0{,}707};$$

$$0 = +y_7 \,.\, 0{,}707 + W + W + W + \frac{W}{2},$$

or
$$y_7 = -\frac{\frac{7}{2}W}{0{,}707}, \qquad \text{and } y_8 = +\frac{\frac{7}{2}W}{0{,}707}.$$

83.] For the strain in flanges we have from Fig. 83,

$$0 = -x_1 \,.\, l \,.\, \text{cot.}\, \alpha + \frac{W}{2} \cdot l \;(\text{rot. } r \,.\, 1);$$

$$x_1 = \frac{\frac{1}{2}W.l}{l \,.\, \text{cotg}\, \alpha}, \qquad \text{or as } \frac{1}{\text{cotg}\, \alpha} = \text{tang}\, \alpha\,,$$

$$x_1 = \tfrac{1}{2}W.\, \text{tang}\, \alpha;$$

and when $W = 10$ tons, $< \varphi = 60°$; therefore $< \alpha = 30°$,
and tang $30° = 0{,}577$ ("Example Stoney");

$$x_1 = 5 \times 0{,}577 = +\, 2{,}9 \text{ tons};$$

or when, for an easier understanding, in Fig. 83,

$$l \,.\, \text{cotg}\, \alpha = h,$$

84.] we have for x_2 in Fig. 84,

$$0 = -x_2 \,.\, h + W.\, l + \frac{W}{2} \cdot 3\, l \;(\text{rot. } r \,.\, 3);$$

$$x_2 = \frac{\frac{5}{2}W.\, l}{h};$$

and as
$$\frac{l}{h} = \text{tang}\, \alpha = 0{,}577,$$

for our example,

$$x_2 = \tfrac{5}{2} \,.\, W \,.\, 0{,}577 = \tfrac{5}{2} \times 10 \times 0{,}577 = 14{,}5 \text{ tons.}$$

85.] So for x_3 in Fig. 85,

$$0 = -x_3 \,.\, h + W.\, l + W.\, 3\, l + \frac{W}{2} \cdot 5\, l \;(\text{rot. } r \,.\, 5);$$

$$x_3 = \tfrac{13}{2} \,.\, W \,.\, 0{,}577 = 37{,}3 \text{ tons},$$

and so further.

86.] For the strain, z, in the lower flanges,

$$0 = +z_1 \,.\, h + \frac{W}{2} \cdot 2\, l \;(\text{rot. } r \,.\, 2 \text{ in Fig. } 86);$$

$$z_1 = -W \cdot \frac{l}{h} = -\, 10 \times 0{,}577 = -\, 5{,}7 \text{ tons.}$$

87.] Fig. 87 gives

$$0 = + z_2 . h + W . 2l + \frac{W}{2} \cdot 4\,l \text{ (rot. } r . 4);$$

$$z_2 = -4W \cdot \frac{l}{h} = -4 \times 10 \times 0{,}577 = -23{,}2 \text{ tons,}$$

In the same way for z_3,

$$0 = + z_3 . h + W . 2\,l + W . 4\,l + \frac{W}{2} \cdot 6 . l;$$

$$z_3 = -9W \cdot \frac{l}{h} = -52 \text{ tons.}$$

88.] In case the load should be connected to the lower apexes (Fig. 88), the equation of equilibrium, per example for z_3, would be

$$0 = + z_3 . h + W . l + W . 3\,l + \frac{W}{2} \cdot 5\,l \text{ (rot. } r . m);$$

$$z_3 = -\tfrac{13}{2} . W \cdot \frac{l}{h} = -37{,}3 \text{ tons;}$$

i. e., the strain is the same as in the flange of the reversed figure, but of opposite character. So also is the strain the same for the other flanges. (See Fig. 88.)

Remark.—In the given example the strains are determined without a certain length for h or l. This is easily explained by the relation which the angle φ or α bears to h and l, as by the extension of one, the other will increase in the same ratio.

[Plates 13, 14 and 15—embracing Figs. 71 to 88.]

D. GIRDERS WITH PARALLEL TOP AND BOTTOM FLANGES.

(*Calculated for a Permanent Load.*)

I. *STRAIN IN DIAGONALS AND VERTICALS.*

The calculation is very similar to the preceding. Provided, again, the load to be connected to the upper or lower apexes for the application of the Method of Moments, we now consider as a special force the reaction of the supports toward the system.

D_1 may represent one-half the weight of loaded truss or the

pressure upon each support (prop), and diminished by the partition of load on this place directly sustained. (Comp. Fig. 21.) The reactive force of support wanted for our calculation will be signified by D.

To compute D, we refer to Fig. 4 in Sect. I., and define first for the following example its numerical value:

Plate 16, Fig. 89.] Through bridge (over-grade bridge), between supports, 48 feet;

8 panels, each 6 feet $= l$;

depth of truss $= 6$ feet $= h$, from centre to centre of top and bottom chords;

the weight of structure $= 3000$ lbs., and the load $= 15000$ lbs.;

gives a permanent load $= 18000$ lbs. per panel;

rolling load $= 0$. (See Sect. II.)

For the distribution of load, see Fig. 89.

Remark.—The strange impression which the arrangement of diagonals, unsymmetrical toward the centre, may first produce, will soon disappear after observation of the advantages for transformation upon succeeding systems.

Whole pressure of truss upon supports $= 8 \times 18000 = 144000$ lbs.;

$D_1 = 18000\,(\frac{1}{8} + \frac{2}{8} + \frac{3}{8} + \frac{4}{8} + \frac{5}{8} + \frac{6}{8} + \frac{7}{8}) + 9000 \times \frac{8}{8} = 72000$ lbs.;

$D = 18000\,(\frac{1}{8} + \frac{2}{8} + \frac{3}{8} + \frac{4}{8} + \frac{5}{8} + \frac{6}{8} + \frac{7}{8}) = 63000$ lbs.

The strain in the post, V_0, is 0, because $x_1 = 0$,

and $$V_8 = -63000.*$$

90.] Excepting the vertical component of y_1 (*i. e.*, $y_1 \sin \varphi$), for a section (Fig. 90), only $D = 63000$ lbs. is a second vertical force.

Both turn to the left around o in the axis x_0; therefore their symbol, —. (Comp. Fig. 22.)

---

* For a deck-bridge (under-grade bridge)—*i. e.*, when the upper apexes are loaded—would be

$$V_0 = -9000, \quad \text{and } V_8 = -72000.$$

The angle of diagonals with a horizontal line will be 45°.

$$0 = -y_1 \sin 45° - D \text{ (rot. } r.o);$$

and as $\sin 45° = 0{,}7$,

$$0 = -y_1 . 0{,}7 - 63000,$$

or $$y_1 = -90000.$$

91.] For V_1 in Fig. 91, only D is acting vertically upon this section.

D turns to the left around o, therefore

$$0 = +V_1 - D \text{ (rot. } r.o);$$

$$0 = V_1 - 63000,$$

or $$V_1 = +63000.$$

92.] According to the preceding, from Fig. 92,

$$0 = -y_2 . 0{,}7 + 18000 - 63000 \text{ (rot. } r.o);$$

$$y_2 = -64285;$$

93.] and from Fig. 93,

$$0 = +V_2 + 18000 - 63000 \text{ (rot. } r.o);$$

$$V_2 = +45000.$$

94.] Fig. 94 gives the equation,

$$0 = -y_3 . 0{,}7 + 18000 + 18000 - 63000$$

$$y_3 = -38570;$$

95.] and from Fig. 95 we have

$$0 = +V_3 + 18000 + 18000 - 63000;$$

$$V_3 = +27000.$$

Now it will be unnecessary to farther accompany the remaining calculations with diagrams.

$$0 = -y_4 . 0{,}7 + 18000 \times 3 - 63000;$$

$$y_4 = -12857;$$

$$0 = +V_4 + 18000 \times 3 - 63000;$$

$$V_4 = +9000;$$

$$0 = -y_5 . 0{,}7 + 18000 \times 4 - 63000;$$

$$y_5 = +12857;$$

$$0 = +V_5 + 18000 \times 4 - 63000;$$

$$V_5 = -9000;$$

$$0 = -y_6 . 0{,}7 + 18000 \times 5 - 63000;$$

$$y_6 = +38570;$$

fig. 89.

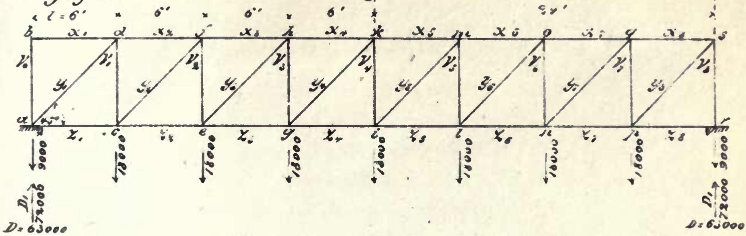


fig 90

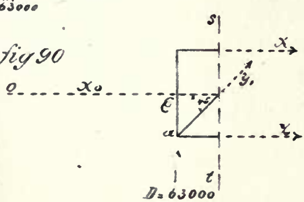


fig. 91

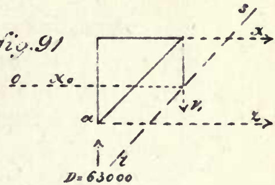


fig. 92

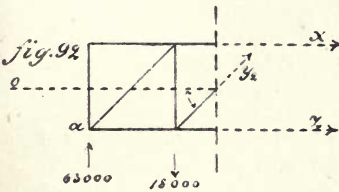


fig. 93

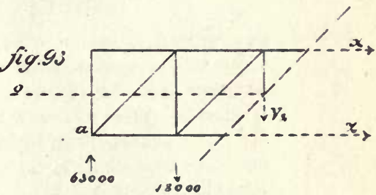


fig. 94.

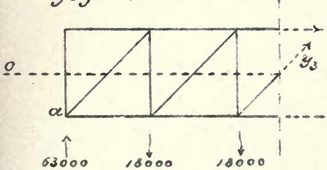


fig. 95

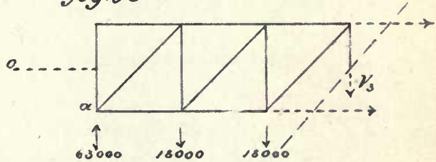


fig. 96

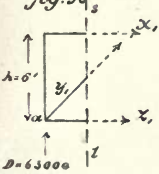


fig. 97

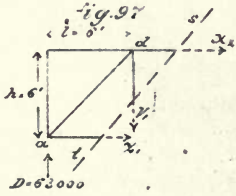


fig. 98.

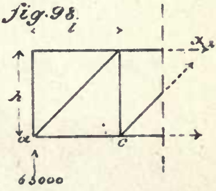


fig. 99

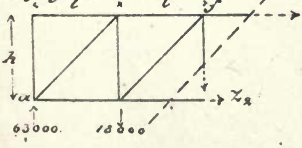
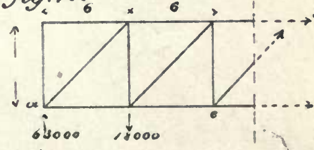


fig. 100



$$0 = + V_6 + 18000 \times 5 - 63000;$$
$$V_6 = - 27000;$$
$$0 = - y_7 . 0{,}7 + 18000 \times 6 - 63000;$$
$$y_7 = + 64285;$$
$$0 = + V_7 + 18000 \times 6 - 63000;$$
$$V_7 = - 45000;$$
$$0 = - y_8 . 0{,}7 + 18000 \times 7 - 63000;$$
$$y_8 = + 90000;$$
$$0 = + V_8 + 18000 \times 7 - 63000;$$
$$V_8 = - 63000.$$

II. *STRAIN IN FLANGES.*

The calculation of strain in flanges differs in so far from the preceding, and will give no difficulty in understanding, as here a suitable point of rotation—*i.e.*, an intersection of those members, separated by a cut, *st*, similar to that one for which we want to compute the strain—is directly presented by the apexes.

For a rough and preliminary control of strain in the top and bottom flanges at the centre of the truss, it is, as 72000 = one-half of the load, 6 feet = depth, and 12 feet = one-fourth of the space between the supports:

$\frac{72000 \times 12}{6} = 144000$ lbs., the horizontal strain in each flange.

(Comp. Sect. I., Fig. 28.)

96.] For Fig. 96 we have the equation of equilibrium,

$$0 = x_1 . h + D . 0 \text{ (rot. in the intersection of } y_1 \text{ and } z_1 \text{ or } a);$$
$$x_1 = 0;$$

97.] and for Fig. 97,

$$0 = - z_1 . h + D . l \text{ (rot. in the intersection of } V_1 \text{ and } x_2 \text{ or } d);$$
$$z_1 = D \cdot \frac{l}{h} = 63000.$$

98.] In the same way for Fig. 98,

$$0 = x_2 . h + D . l \text{ (rot. } r . c);$$
$$0 = x_2 . 6 + 63000 \times 6;$$
$$x_2 = - 63000;$$

99.] and for Fig. 99,

$$0 = -z_2 . h - 18000 . l + D . 2l \text{ (rot. } r . f);$$
$$0 = -z_2 . 6 - 18000 \times 6 + 63000 \times 12;$$
$$z_2 = +108000.$$

100.] $0 = x_3 . 6 - 18000 \times 6 + 63000 \times 12$ (rot. $r . e$, Fig. 100);

$$x_3 = -108000.$$

Plate 17, Fig. 101.] $0 = -z_3 . 6 - 18000 (6 + 12) + 63000 \times 18$ (rot. $r . h$, Fig. 101);

$$z_3 = +135000.$$

In the same way for the other flanges,

$$0 = x_4 . 6 - 18000 (6 + 12) + 63000 \times 18 \text{ (rot. } r . g, \text{ Fig 89)};$$
$$x_4 = -135000;$$
$$0 = -z_4 . 6 - 18000 (6 + 12 + 18) + 63000 \times 24 \text{ (rot. } r . k);$$
$$z_4 = +144000;$$
$$0 = x_5 . 6 - 18000 (6 + 12 + 18) + 63000 \times 24 \text{ (rot. } r . i);$$
$$x_5 = -144000;$$
$$0 = -z_5 . 6 - 18000 (6 + 12 + 18 + 24) + 63000 \times 30 \text{ (rot. } r . m);$$
$$z_5 = +135000;$$
$$0 = x_6 . 6 - 18000 \times 60 + 63000 \times 30 \text{ (rot. } r . l);$$
$$x_6 = -135000;$$
$$0 = -z_6 . 6 - 18000 \times 90 + 63000 \times 36 \text{ (rot. } r . o);$$
$$z_6 = +108000;$$
$$0 = -x_7 . 6 - 180000 \times 90 + 63000 \times 36 \text{ (rot. } r . n);$$
$$x_7 = -108000;$$
$$0 = -z_7 . 6 - 18000 \times 126 + 63000 \times 42 \text{ (rot. } r . q);$$
$$z_7 = +63000;$$
$$0 = -x_8 . 6 - 18000 \times 126 + 63000 \times 42 \text{ (rot. } r . p);$$
$$x_8 = 63000;$$
$$0 = -z_8 . 6 - 18000 \times 168 + 63000 \times 48 \text{ (rot. } r . s);$$
$$z_8 = 0.$$

102.] From the calculation of the first panel for a reversed system (Fig. 102), it will be perceived that here the results are the same as before, and so also for the other panels:

$$0 = + y_1 . \sin 45^\circ - D \text{ (rot. } r . o);$$
$$0 = y_1 . 0{,}7 - 63000;$$
$$y_1 = \frac{63000}{0{,}7} = + 90000;$$
$$0 = + x_1 . h + D . l \text{ (rot. in the intersection of } y_1 \text{ and } z_1, \text{ or } c);$$
$$0 = x_1 . 6 + 63000 \times 6;$$
$$x_1 = - 63000;$$
$$0 = - z_1 . h + D . 0 \text{ (rot. in the intersection of } y_1 \text{ and } x_1, \text{ or } b);$$
$$z_1 = 0.$$

103.] The results of the foregoing are combined in Fig. 103, the compressive strains being represented by double lines.

III. *TRANSFORMATIONS.*

104.] Truss symmetrical to the centre, with vertical tie-rods and oblique braces. (Howe's system, Fig. 104.)

105.] Pratt truss (Fig. 105), with oblique tie-rods and vertical braces, in which the vertical anchors, *a*, *b*, on the abutment can be spared, the same as in Fig. 114.*

IV. *GENERAL REMARKS.*

106.] For Howe's system (deck-bridge, Fig. 106), there will be in the vertical end post a compression = 9000 lbs., at the centre post 0, and for the reversed system at the centre post, 18000.

Further specification of this and the reversed and combined system, also for the isometrical truss, will be found in the connected tables (Figs. 108 to 117), forming thus a useful guide, especially for the strain in flanges.

The strain in flanges being a maximum for a full load is to be calculated always for a full load, also in the case of a combined (permanent and rolling) load, and it will be treated accordingly in the following pages.

The influence of a *rolling* load upon braces and tie-rods, being so far unconsidered, is very essential, as there is at the centre a tendency to horizontal dislocation, which is prevented in the Howe

* Keystone Bridge Company, Pittsburg.

truss by counter-braces, in the isometrical truss by counter-rods, and in other bridges by panel-rods or braces.

In regard to the connecting of a load, we will only remark that for both permanent and rolling loads it is quite the same for the diagonals and horizontal flanges whether the load is connected on the top (deck-bridge) or on the bottom (through-bridge) or to the vertical posts.

But for the post itself in the last case we have to consider the thesis, that

The vertical component of diagonals is equal to the strain in verticals—only of opposite character—when they meet at an unloaded point.

When, therefore, the load is connected between the top and bottom of the vertical brace, both top and bottom apexes are unloaded points; hence the strain in the upper and lower part of the vertical is the vertical strain (vertical component) of the adjoining diagonal, only of opposite character.

So, for example, would be for the post V_7, in Fig. 105, in the lower part,

$$-\ 90000 \,.\, \sin 45°,$$

or

$$-\ 90000 \times 0{,}7 = -\ 63000,$$

which is the vertical component of the adjoining diagonal, the symbol being reversed.

107.] In the upper part would be (Fig. 107),

$$-\ 64285 \times 0{,}7 = -\ 45000 \text{ lbs. (according to the same thesis).}$$

In general, we can derive the strain in the verticals from the diagonals without difficulty. So for the same post, V_7, from Fig. 105,

$$V_7 = -\ y_7 \,.\, \cos 45 = -\ 64285 \times 0{,}7 = -\ 45000,$$

and

$$V_6 = -\ 38570 \times 0{,}7 = -\ 27000;$$

$$V_5 = -\ 12857 \times 0{,}7 = -\ 9000.$$

DIRECTIONS FOR THE CALCULATION OF COMPLEX SYSTEMS.

For complex systems, as shown in the post bridge,* the Linville, etc. (Fig. 107ᵃ to 107ᶜ), the calculation may be made for each system separately, to derive the strain in braces and tie-rods. For

* Atlantic Bridge Works, New York.

fig. 101.

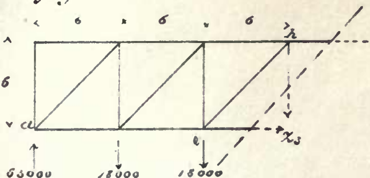


fig. 102

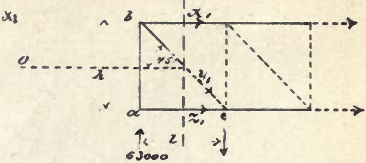


fig. 103

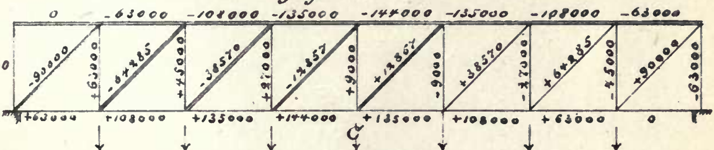


fig. 104

fig. 105

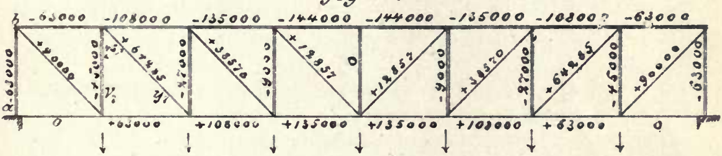


fig. 106

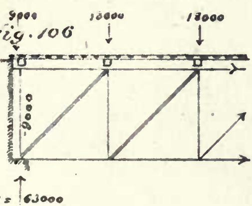


fig. 107

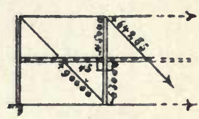


fig. 107a

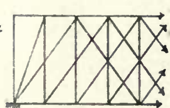


fig. 107b

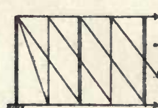
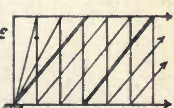


fig. 107c



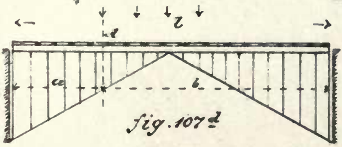


fig. 107d

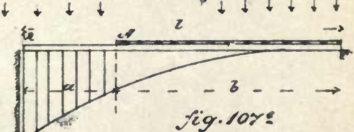


fig. 107e

the strain in flanges, in laying the skeletons together, the covering parts of the flanges can be added, as shown in Sect. II. on the isometrical truss, Fig. 146.*

In the same way, by observation of the angles, this method can be applied to lattice systems.

When, in Fig. 107^d, q is the weight per unit of length (foot), so,

107^c.] for an equally-distributed load, is $\frac{q \cdot l}{2}$ the shearing strain on the abutment, and at a point, A, the shearing strain =

$$q \cdot \frac{a+b}{2} - q \cdot a = \frac{q}{2}(b-a);$$

but at the centre = 0.

107^d.] When the greater segment of a girder is loaded (Fig. 107^e), the shearing strain at a point, A, is

$$q \cdot \frac{b^2}{2 \cdot l} = \text{the reaction, } R\text{, of the abutment.}$$

This is the shearing strain throughout the segment a, and increases, when the load moves forward, in the same ratio as the ordinates of a parabola.

Subtracting the former quantity from the latter—*i. e.*,

$$\frac{q \cdot b^2}{2 \cdot l} - \frac{q}{2} \cdot (b-a) = q \cdot \frac{a^2}{2 \cdot l},$$

showing that the shearing strain is greater when, instead of the whole girder, the greater segment is loaded; and this quantity is the shearing strain throughout the segment b when the shorter segment, a, is loaded. (Mr. Stoney.)†

[Plates 16 and 17—embracing Figs. 89 to 107.]

---

* Isometrical Truss Bridge Company, Pottstown, Pa.

† Mr. Shreve, in Van Nostrand's "Eclectic Engineering Magazine," No. xx., August, 1870; Vol. III., page 193.

E. COMPARATIVE TABLES OF RESULTING STRAINS FOR A PERMANENT LOAD.

(*To Derive the Strains without Special Calculation.*)

I. *SYSTEM OF RIGHT-ANGLED TRIANGLES.*

a. *Single System*—oblique braces, vertical tie-rods, Fig. 108. (Howe truss.)

1. Through Bridge (over-grade bridge).

p signifies the load and weight for one-half panel; therefore $2p$ is the vertical strain at each apex.

For a full load, denoting the number of panels in one-half of the girder with n (here $= 5$), and the respective panels from centre of truss toward the end with x,

$$H_0 = +2p; \qquad H_x = +(2x+1).p; \qquad H_n = o;$$

$$D_x = -(2x-1)\,p\cdot\frac{d}{h}; \qquad S_x = +[n^2-(x-1)^2]\,p\cdot\frac{b}{h};$$

$$P_x = -S(x+1) = -(n^2-x^2)\,p\cdot\frac{b}{h};$$

2. Deck Bridge (under-grade bridge).

Plate 18, Fig. 108. Here the tension in the vertical connections (tie-rods) will be diminished each $2p$; *i. e.*, in the skeleton (Fig. 108),

$$H_0 = o; \qquad H = +p; \qquad H_2 = +3p; \qquad H_3 = +5p, \text{ etc.}$$

The vertical post on the abutment will have the compression $= -p$.

In the strain in flanges and oblique braces there will be no difference.

b. *Single System*—vertical braces, oblique tie-rods. (Fig. 109.)

1. Through Bridge.

For a full load,

$$H_0 = o; \qquad H_x = -(2x-1).p;$$

$$D_x = +(2x-1)\,p\cdot\frac{d}{h}; \qquad p_x = -[n^2-(x-1)^2]\,p\cdot\frac{b}{h};$$

$$S_x = + P(x+1) = + (n^2 - x^2)\, p \cdot \frac{b}{h} \cdot$$

2. Deck Bridge.

109.] The strain in the vertical braces will be increased each $-2p$, so that in the skeleton (Fig. 109),

$$H_0 = -2p; \qquad H_1 = -3p; \qquad H_2 = -5p, \text{ and so on.}$$

The compression in the vertical post on the abutment will be increased by p; therefore

$$H_5 = -10p.$$

c. *Combined System.*

110ᵃ.] 1. For a full load by a through bridge (Fig. 110ᵃ), the tension in the vertical connections all the same, or

$$H_x = +p.$$

The vertical post on the abutment has the compression,

$$H_n = -(n - \tfrac{1}{2})\, p.$$

For any one panel the tension and compression in the crossing diagonals, and also the strain in the upper or lower flange, is the same—*i. e.*,

$$D_x = \pm (x - \tfrac{1}{2})\, p \cdot \frac{d}{h};$$

$$P_x = S_x = \pm \left(n^2 - \frac{x^2 + (x-1)^2}{2} \right) \cdot p \cdot \frac{b}{h} \cdot$$

2. For a deck-bridge changes the tension $+p$ in the vertical connections to compression, or $-p$.

110ᵃ.] The compression in the vertical post on the abutment will be increased by $-p$; therefore in the above skeleton,

$$H_5 = -5\tfrac{1}{2} \cdot p.$$

3. For the system of right-angled triangles, when the connecting

110ᵃ.] of cross-beams is not on the top or bottom, but between both on the verticals, the calculation for the part of the vertical above the connection is the same as under 1, and for the lower part of the vertical beneath the connection, the same as under 2.

For the rest the calculation is as under 1.

II. *SYSTEM OF ISOSCELES BRACING.*

(*Load Permanent and Equally Distributed.*)

a. *Single System* (triangular truss).

Plate 19, Fig. 111. 1. A loaded apex at the centre. (Fig. 111.)

The number of panels extended as occasion requires.

In general, each pair of diagonals from centre to abutment increases by $W \cdot \frac{d}{h}$ (*i. e.*, $W \sec \alpha$).

112.] 2. An unloaded apex at the centre. (Fig. 112.)

Each pair of diagonals from centre to abutment increases by $W \cdot \frac{d}{h}$.

When each diagonal represents one panel $= b$, and the number of panels $= n$ for one-half of the truss, counting from centre to
113, 114.] abutment, there results the strain in flanges for an even n, say $n = 6$, and a loaded apex at the centre (Fig. 113), or an unloaded apex at the centre (Fig. 114).

For an uneven n, say $n = 7$, and a loaded apex at the centre
115, 116.] (see Fig. 115); and for an uneven n, say $n = 7$, but an unloaded apex at the centre (see Fig. 116).

b. *Combined System* (isometrical truss).

117.] A loaded apex at the centre, and $n = 6$ (Fig. 117).

For a full load in the xth panel the tension in the tie-rod or $D_x = + x \cdot \frac{W}{4} \cdot \frac{d}{h}$, and the compression of diagonal, $D_x = -(x-1)\frac{W}{4} \cdot \frac{d}{h}$.

The strains for this figure (where $n = 6$) will also be obtained by laying the figures 113 and 114 (A and B) together, making there $\frac{W}{4}$, instead of $\frac{W}{2}$, and adding the strains in flanges where they cover each other.

[Plates 18 and 19—embracing Figs. 108 to 117.]

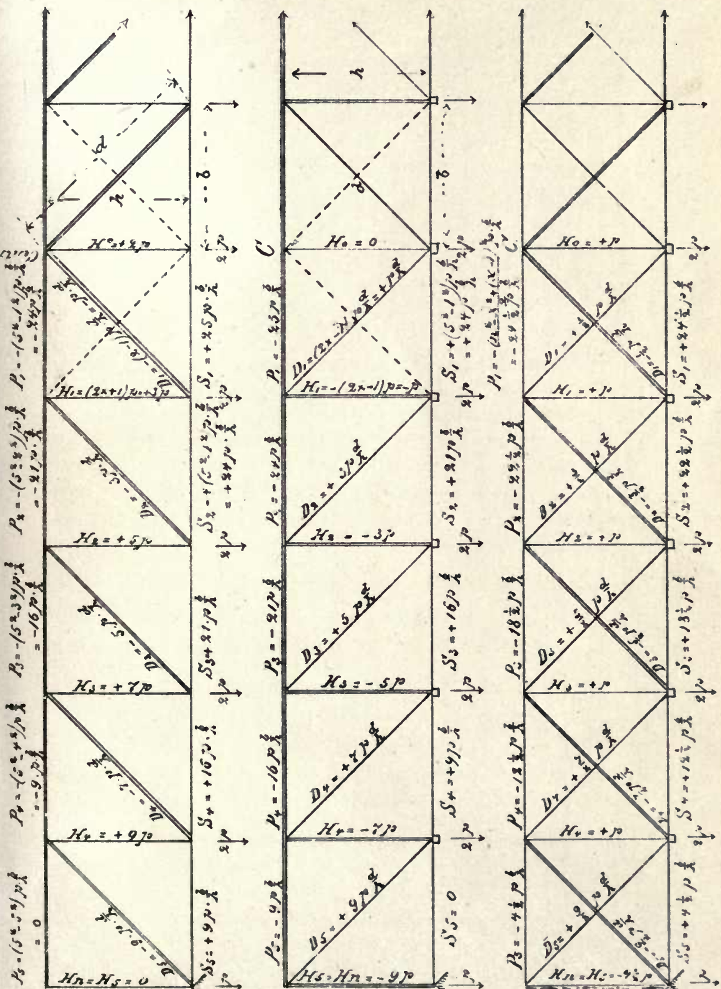


fig. 108

fig. 109

fig. 110

fig.111.

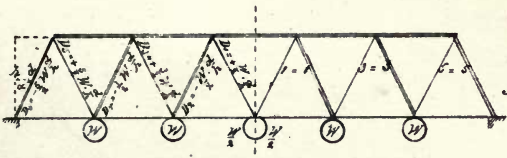


fig.112.

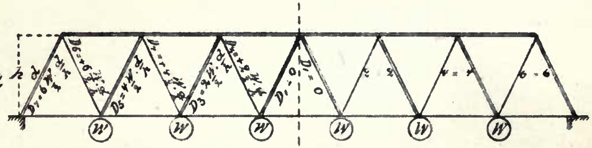


fig.113

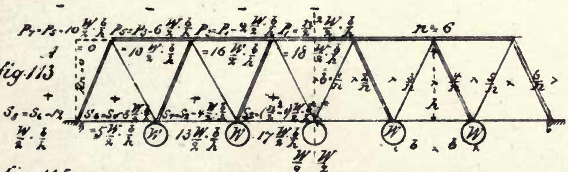


fig.114

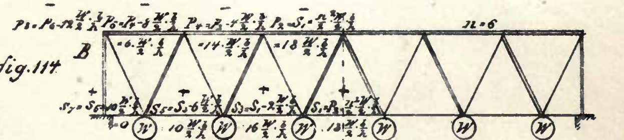


fig.115

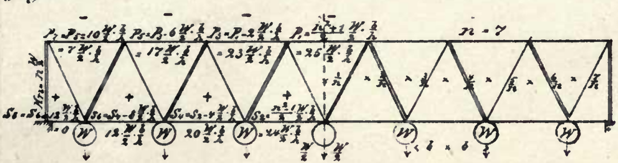


fig.116

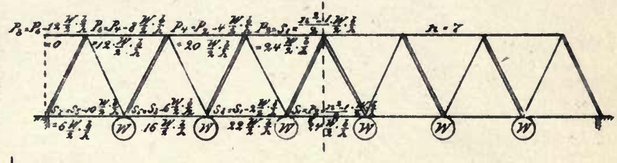
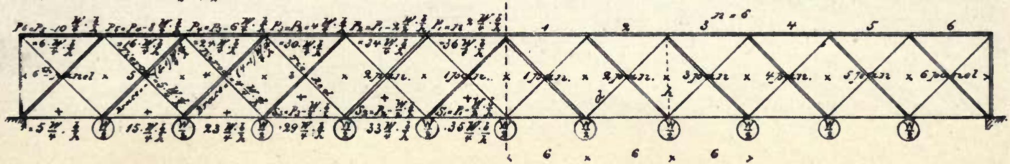


fig.117



SECTION II.

GIRDERS CALCULATED FOR COMBINED (PERMANENT AND ROLLING) LOAD.

A. GIRDERS WITH PARALLEL TOP AND BOTTOM FLANGES.

I. *THE RIGHT-ANGLED SYSTEM.*

For the strain in flanges I refer to the preceding. It needs no repetition, as the strain in flanges is the largest for a full (permanent) load. (Figs. 108 to 117.)

As mentioned at the close of Sect. I. (General Remarks), the influence of a rolling load upon braces and tie-rods is very essential, and it is greatest when a traversing train of uniform density reaches the centre of the bridge.

Plate 20, Fig. 118. For explanation we choose the diagonal y_3 in Fig. 118. q is the rolling load (traversing train) for one panel; p is the permanent load (weight of structure); so, again, is (see Figs. 81 and 94),

$$0 = -y_3 . \sin\varphi - D . \infty + \left(\frac{p}{2} + \frac{q}{2}\right)\infty + (p+q)\infty + (p+q)\infty,$$

or I. $0 = -y_3 . \sin\varphi - D + (p+q) + (p+q) + \left(\dfrac{p}{2} + \dfrac{q}{2}\right)$

(rot. $r . o$).

As before, $y_3 . \sin\varphi$ is the vertical component of y_3, and D is the reaction of abutment, or

$$D_1 = (p+q)\left(\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8} + \tfrac{4}{8} + \tfrac{5}{8} + \tfrac{6}{8} + \tfrac{7}{8} + \tfrac{4}{8}\right). \quad \text{(Fig. 119.)}$$

II. $D = (p+q)\left(\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8} + \tfrac{4}{8} + \tfrac{5}{8} + \tfrac{6}{8} + \tfrac{7}{8}\right)$. (See Fig. 89.)

We include the value of D_1 in Equation I., so that the influence

of each fraction of rolling load and permanent load upon D may be visible in its proper place.

III. $0 = - y_3 . \sin \varphi - (p + q) \left[\left(\frac{1}{8} + \frac{2}{8} + \frac{3}{8} + \frac{4}{8} + \frac{5}{8}\right) + \left(\frac{6}{8} - 1\right) + \left(\frac{7}{8} - 1\right) + \left(\frac{4}{8} - \frac{1}{2}\right)\right]$,

or $0 = - y_3 . \sin \varphi - (p + q) \left[\left(\frac{1}{8} + \frac{2}{8} + \frac{3}{8} + \frac{4}{8} + \frac{5}{8}\right) - \left(1 - \frac{6}{8}\right) - \left(1 - \frac{7}{8}\right)\right]$;

and now, separating the members containing the permanent load, p, from those containing the rolling load, q, and separating also the positive and negative members of the movable load,

$$0 = - y_3 . \sin \varphi - p \left[\frac{1}{8} + \frac{2}{8} + \frac{3}{8} + \frac{4}{8} + \frac{5}{8} - \left(1 - \frac{6}{8}\right) - \left(1 - \frac{7}{8}\right)\right];$$
$$- q \left(\frac{1}{8} + \frac{2}{8} + \frac{3}{8} + \frac{4}{8} + \frac{5}{8}\right) + q \left[\left(1 - \frac{6}{8}\right) + \left(1 - \frac{7}{8}\right)\right].$$

In this equation may be neglected at one time the positive part, and at another time the negative part, of the movable load, q, and the maximum and minimum strain for y will be obtained, viz.:

I. $0 = - y_3 . \sin \varphi - p \left(\frac{15}{8} - \frac{2}{8} - \frac{1}{8}\right) - q . \frac{15}{8}$ (max. compr.).

II. $0 = - y_3 . \sin \varphi - p \left(\frac{15}{8} - \frac{2}{8} - \frac{1}{8}\right) + q \left(\frac{2}{8} + \frac{1}{8}\right)$ (min. compr.),

or when the factor of q is greater than the factor of p, then it will be the maximum tension in this diagonal.

In Equation III. the fraction to the right is

$$(p + q) \left(\frac{4}{8} - \frac{1}{2}\right) = 0;$$

120.] therefore we introduce again D instead of D_1, and find from Fig. 120, under the same supposition of $x = \infty$ for the vertical rod V_3,

$$0 = + V_3 \infty - D \infty + (p + q) \infty,$$

or $$0 = + V_3 - D + (p + q) + (p + q),$$

differing from the equation for y only in so far as here is V instead of $- y \sin \varphi$.

Example.—Calculation of diagonals and verticals for a combined load:

121.] Length of truss $= 48$ feet; (Fig. 121.)
8 panels, each 6 feet $= l$;
height $= 6$ feet $= h$;
permanent load $= 3000$ lbs. per panel $= p$;
rolling load $= 15000$ lbs. per panel $= q$.

The load being connected to the lower apexes, the pressure, D, of supports from Fig. 89 = 63000 lbs.

122.] Fig. 122 gives the equations,

$$0 = -y_1 . \sin 45 - D \text{ (rot. } r.o);$$

$$0 = -y_1 . 0,7 - 63000;$$

or $$y_1 = -\frac{63000}{0,7} = -90000.$$

123.] From Fig. 123 we have

$$0 = -y_2 . 0,7 - D + (p + q),$$

and including D combined with $p + q$ on its proper place,

$$0 = -y_2 . 0,7 - (p + q) \left[\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8} + \tfrac{4}{8} + \tfrac{5}{8} + \tfrac{6}{8} + (\tfrac{7}{8} - 1)\right],$$

or $0 = -y_2 . 0,7 - (p + q) \left[\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8} + \tfrac{4}{8} + \tfrac{5}{8} + \tfrac{6}{8} - (1 - \tfrac{7}{8})\right];$

$$0 = -y_2 . 0,7 - p\,(\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8} + \tfrac{4}{8} + \tfrac{5}{8} + \tfrac{6}{8} - \tfrac{1}{8}) - q\,(\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8} + \tfrac{4}{8} + \tfrac{5}{8} + \tfrac{6}{8}) - q\,(-\tfrac{1}{8});$$

$$0 = -y_2 . 0,7 - p . \tfrac{20}{8} - q . \tfrac{21}{8} + q . \tfrac{1}{8},$$

and from this the two equations,

1.) $0 = -y_2 . 0,7 - p . \tfrac{20}{8} - q . \tfrac{21}{8}$, or $y_2 . 0,7 = -3000 \times \tfrac{20}{8} - 15000 \times \tfrac{21}{8}$.

2.) $0 = -y_2 . 0,7 - p . \tfrac{20}{8} + q . \tfrac{1}{8}$, or $y_2 . 0,7 = -3000 \times \tfrac{20}{8} + 15000 \times \tfrac{1}{8}$.

I. $y_2 = -66964$ (max. compr.);

II. $y_2 = -8036$ (min. compr.).

124.] Fig. 124 gives

$$0 = -y_3 . 0,7 - D + (p + q) + (p + q);$$

$$0 = -y_3 . 0,7 - (p + q) \left[\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8} + \tfrac{4}{8} + \tfrac{5}{8} + (\tfrac{6}{8} - 1) + (\tfrac{7}{8} - 1)\right];$$

$$0 = -y_3 . 0,7 - (p + q) \left[\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8} + \tfrac{4}{8} + \tfrac{5}{8} - (1 - \tfrac{6}{8}) - (1 - \tfrac{7}{8})\right];$$

$$0 = -y_3 . 0,7 - (p + q)\,(\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8} + \tfrac{4}{8} + \tfrac{5}{8} - \tfrac{2}{8} - \tfrac{1}{8});$$

$$0 = -y_3 . 0,7 - p . \tfrac{12}{8} - q . \tfrac{15}{8} + q . \tfrac{3}{8};$$

1.) $0 = -y_3 . 0,7 - p . \tfrac{12}{8} - q . \tfrac{15}{8}$;

2.) $0 = -y_3 . 0,7 - p . \tfrac{12}{8} + q . \tfrac{3}{8}$.

$$\text{I. } y_3 = -46608;$$
$$\text{II. } y_3 = +1608.$$

For further equations, sketches will not be necessary, and they may be written in reduced forms.

$$0 = -y_4 . 0{,}7 - D + (p+q) + (p+q) + (p+q);$$
$$0 = -y_4 . 0{,}7 - (p+q)\left(\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8} + \tfrac{4}{8} - \tfrac{3}{8} - \tfrac{2}{8} - \tfrac{1}{8}\right);$$
$$0 = -y_4 . 0{,}7 - p . \tfrac{4}{8} - q . \tfrac{10}{8} + q . \tfrac{6}{8}.$$
$$1.)\ 0 = -y_4 . 0{,}7 - p . \tfrac{4}{8} - q . \tfrac{10}{8};$$
$$2.)\ 0 = -y_4 . 0{,}7 - p . \tfrac{4}{8} + q . \tfrac{6}{8}.$$
$$\text{I. } y_4 = +13930;$$
$$\text{II. } y_4 = -28930.$$

$$0 = -y_5 . 0{,}7 - D + (p+q) + (p+q) + (p+q) + (p+q) + (p+q);$$
$$0 = -y_5 . 0{,}7 - (p+q)\left(\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8} - \tfrac{4}{8} - \tfrac{3}{8} - \tfrac{2}{8} - \tfrac{1}{8}\right);$$
$$0 = -y_5 . 0{,}7 + p . \tfrac{4}{8} - q . \tfrac{6}{8} + q . \tfrac{10}{8}.$$
$$1.)\ 0 = -y_5 . 0{,}7 + p . \tfrac{4}{8} - q . \tfrac{6}{8};$$
$$2.)\ 0 = -y_5 . 0{,}7 + p . \tfrac{4}{8} + q . \tfrac{10}{8}.$$
$$\text{I. } y_5 = -13930;$$
$$\text{II. } y_5 = +28930.$$

$$0 = -y_6 . 0{,}7 - (p+q)\left(\tfrac{1}{8} + \tfrac{2}{8} - \tfrac{5}{8} - \tfrac{4}{8} - \tfrac{3}{8} - \tfrac{2}{8} - \tfrac{1}{8}\right);$$
$$0 = -y_6 . 0{,}7 + p . \tfrac{12}{8} - q . \tfrac{3}{8} + q . \tfrac{15}{8}.$$
$$1.)\ 0 = -y_6 . 0{,}7 + p . \tfrac{12}{8} - q . \tfrac{3}{8};$$
$$2.)\ 0 = -y_6 . 0{,}7 + p . \tfrac{12}{8} + q . \tfrac{15}{8}.$$
$$\text{I. } y_6 = -1608;$$
$$\text{II. } y_6 = +46608.$$

$$0 = -y_7 . 0{,}7 - (p+q)\left(\tfrac{1}{8} - \tfrac{6}{8} - \tfrac{5}{8} - \tfrac{4}{8} - \tfrac{3}{8} - \tfrac{2}{8} - \tfrac{1}{8}\right);$$
$$0 = -y_7 . 0{,}7 + p . \tfrac{20}{8} - q . \tfrac{1}{8} + q . \tfrac{21}{8}.$$
$$1.)\ 0 = -y_7 . 0{,}7 + p . \tfrac{20}{8} - q . \tfrac{1}{8};$$
$$2.)\ 0 = -y_7 . 0{,}7 + p . \tfrac{20}{8} + q . \tfrac{21}{8}.$$

$$\text{I. } y_7 = +8036;$$
$$\text{II. } y_7 = +66964.$$

$$0 = -y_8 \,.\, 0{,}7 - (p+q)\left(-\tfrac{7}{8} - \tfrac{6}{8} - \tfrac{5}{8} - \tfrac{4}{8} - \tfrac{3}{8} - \tfrac{2}{8} - \tfrac{1}{8}\right);$$
$$0 = -y_8 \,.\, 0{,}7 + p \,.\, \tfrac{28}{8} + q \,.\, \tfrac{28}{8}.$$
$$1.)\ y_8 = \frac{p+q}{0{,}7} \times \tfrac{28}{8}.$$
$$\text{I. } y_8 = +90000.$$

In the preceding examples, as already stated, the value of vertical V can be easily obtained from the strain in the diagonals; so for V_3, making the character (symbol) reversed.

$$V_3 = +y_3 \,.\, 0{,}7 = 46608 \times 0{,}7 = +32625,$$

and $$V_3 = -y_3 \,.\, 0{,}7 = -1608 \times 0{,}7 = -1125;$$

but the usual way is here preferable.

$$V_0 = 0;$$
$$V_8 = -63000;$$

and for a deck-bridge would be

$$V_0 = -9000;$$
$$V_8 = -72000.$$

125.] $$0 = V_1 - D \text{ (rot. } r \,.\, o); \qquad \text{(Fig. 125.)}$$
$$V_1 = +63000.$$

126.] $$0 = V_2 - D + (p+q); \qquad \text{(Fig. 126.)}$$
$$0 = V_2 - (p+q)\left[\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8} + \tfrac{4}{8} + \tfrac{5}{8} + \tfrac{6}{8} + \left(\tfrac{7}{8} - 1\right)\right];$$
$$0 = V_2 - (p+q)\left[\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8} + \ldots + \tfrac{6}{8} - \left(1 - \tfrac{7}{8}\right)\right];$$
$$0 = V_2 - (p+q)\left(\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8} + \ldots + \tfrac{6}{8} - \tfrac{1}{8}\right);$$
$$0 = V_2 - p \,.\, \tfrac{20}{8} - q \,.\, \tfrac{21}{8} + q \,.\, \tfrac{1}{8}.$$

1.) $0 = V_2 - p \,.\, \tfrac{20}{8} - q \,.\, \tfrac{21}{8}$, or $V_2 = -3000 \times \tfrac{20}{8} - 15000 \times \tfrac{21}{8}$;

2.) $0 = V_2 - p \,.\, \tfrac{20}{8} + q \,.\, \tfrac{1}{8}$, or $V_2 = 3000 \times \tfrac{20}{8} + 15000 \times \tfrac{1}{8}$.

$$\text{I. } V_2 = +46875 \text{ (max. tension)};$$
$$\text{II. } V_2 = +5625.$$

127.] $$0 = V_3 - D + (p+q) + (p+q); \quad \text{(Fig. 127.)}$$

$$0 = V_3 - (p+q)\left[\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8} + \tfrac{4}{8} + \tfrac{5}{8} + (\tfrac{6}{8} - 1) + (\tfrac{7}{8} - 1)\right];$$

$$0 = V_3 - (p+q)\left[\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8} + \tfrac{4}{8} + \tfrac{5}{8} - (1 - \tfrac{6}{8}) - (1 - \tfrac{7}{8})\right];$$

$$0 = V_3 - p\,(\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8} + \tfrac{4}{8} + \tfrac{5}{8} - \tfrac{2}{8} - \tfrac{1}{8}) - q\,(\tfrac{1}{8} + \tfrac{2}{8} + \ldots + \tfrac{5}{8}) + q\,(\tfrac{2}{8} + \tfrac{1}{8});$$

$$0 = V_3 - p \cdot \tfrac{12}{8} + q \cdot \tfrac{15}{8} - q \cdot \tfrac{3}{8}.$$

1.) $0 = V_3 - p \cdot \tfrac{12}{8} + q \cdot \tfrac{15}{8}$;

2.) $0 = V_3 - p \cdot \tfrac{12}{8} - q \cdot \tfrac{3}{8}$.

I. $V_3 = +\,32625$ (max. tension).

II. $V_3 = -\,1125$ (max. compr.).

$$0 = V_4 - D + (p+q) + (p+q) + (p+q);$$

$$0 = V_4 - (p+q)\ \tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8} + \tfrac{4}{8} - \tfrac{3}{8} - \tfrac{2}{8} - \tfrac{1}{8});$$

$$0 = V_4 - p \cdot \tfrac{4}{8} - q \cdot \tfrac{10}{8} + q \cdot \tfrac{6}{8}.$$

1.) $0 = V_4 - p \cdot \tfrac{4}{8} - q \cdot \tfrac{10}{8}$;

2.) $0 = V_4 - p \cdot \tfrac{4}{8} + q \cdot \tfrac{6}{8}$.

I. $V_4 = +\,20250$;

II. $V_4 = -\,9750$.

$$0 = V_5 - D + (p+q) + (p+q) + (p+q) + (p+q);$$

$$0 = V_5 - (p+q)\,(\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8} - \tfrac{4}{8} - \tfrac{3}{8} - \tfrac{2}{8} - \tfrac{1}{8});$$

$$0 = V_5 + p \cdot \tfrac{4}{8} - q \cdot \tfrac{6}{8} + q \cdot \tfrac{10}{8}.$$

1.) $0 = V_5 + p \cdot \tfrac{4}{8} + q \cdot \tfrac{10}{8}$;

2.) $0 = V_5 + p \cdot \tfrac{4}{8} - q \cdot \tfrac{6}{8}$.

I. $V_5 = -\,20250$;

II. $V_5 = +\,9750$.

$$0 = V_6 - (p+q)\,(\tfrac{1}{8} + \tfrac{2}{8} - \tfrac{5}{8} - \tfrac{4}{8} - \tfrac{3}{8} - \tfrac{2}{8} - \tfrac{1}{8});$$

$$0 = V_6 + p \cdot \tfrac{12}{8} - q \cdot \tfrac{3}{8} + q \cdot \tfrac{15}{8};$$

1.) $0 = V_6 + p \cdot \tfrac{12}{8} - q \cdot \tfrac{3}{8}$;

2.) $0 = V_6 + p \cdot \tfrac{12}{8} + q \cdot \tfrac{15}{8}$.

fig. 118

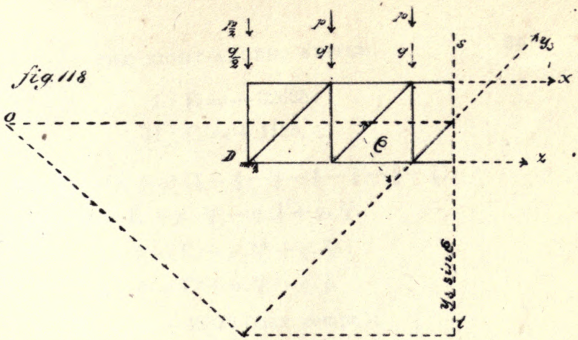


fig. 119

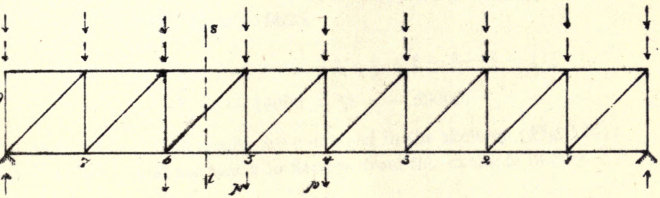


fig. 120

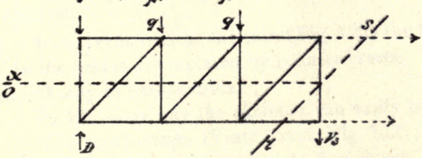


fig. 121

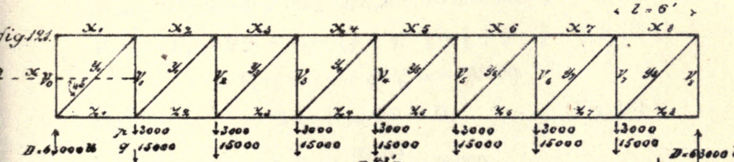


fig. 122

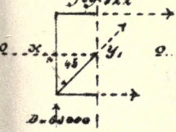


fig. 123

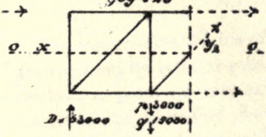


fig. 124

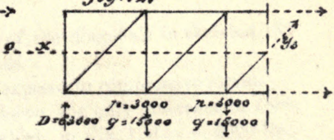


fig. 125

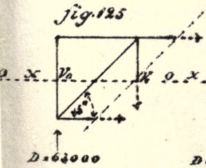


fig. 126

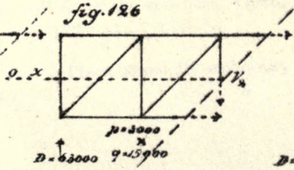
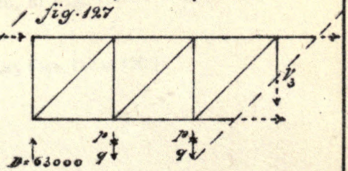


fig. 127



$$\text{I. } V_6 = -32625;$$
$$\text{II. } V_6 = +1125.$$

$$0 = V_7 - (p+q)\left(\tfrac{1}{8} - \tfrac{6}{8} - \tfrac{5}{8} - \tfrac{4}{8} - \tfrac{3}{8} - \tfrac{2}{8} - \tfrac{1}{8}\right);$$
$$0 = V_7 + p \cdot \tfrac{20}{8} - q \cdot \tfrac{1}{8} + q \cdot \tfrac{21}{8}.$$
$$1.)\ 0 = V_7 + p \cdot \tfrac{20}{8} + q \cdot \tfrac{21}{8};$$
$$2.)\ 0 = V_7 + p \cdot \tfrac{20}{8} - q \cdot \tfrac{1}{8}.$$
$$\text{I. } V_7 = -46875 \text{ (max. compr.)};$$
$$\text{II. } V_7 = -5625.$$

$$0 = V_8 - (p+q)\left(-\tfrac{7}{8} - \tfrac{6}{8} - \tfrac{5}{8} - \tfrac{4}{8} - \tfrac{3}{8} - \tfrac{2}{8} - \tfrac{1}{8}\right);$$
$$V_8 = -18000 \times \tfrac{28}{8} = -63000.$$

Plate 21, Fig. 128.] The results are combined in the skeleton (Fig. 128); and the strain in flanges, from the example in Sect. I., D., is repeated.

129.] In the symmetrical skeleton (Fig. 129) the tensile strains in the braces are replaced by counter-braces. The verticals are exposed only to tensile strain.

130.] In the same way the skeleton can easily be transformed to a reversed system (Pratt Truss, Fig. 132); but the uniformity in calculation may here, also, be first shown. (Fig. 130.)

$$V_8 = -D, \text{ or } 0 = -V_8 - (p+q)\left(\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8} + \tfrac{4}{8} + \tfrac{5}{8} + \tfrac{6}{8} + \tfrac{7}{8}\right);$$
$$V_8 = -(p+q)\tfrac{28}{8} = -63000.$$

131.] $$0 = +y_8 \cdot 0{,}7 - D; \qquad \text{(Fig. 131.)}$$

$$y_8 = +\frac{63000}{0{,}7} = 90000.$$

132.] The compressive strains of the diagonals in skeleton 132 are replaced by counter-rods.

The verticals or posts are only exposed to compressive strain.

For a deck-bridge, as before shown, the only difference in compression will be in the end-post, which, in Fig. 129 = — 9000 lbs., and in Fig. 132 = — 72000 lbs.

[Plates 20 and 21—embracing Figs. 118 to 132.]

II. *ISOSCELES BRACING.*

1. TRIANGULAR TRUSS.

Plate 22, Fig. 133.] In Fig. 133, the load on the bottom is sustained one-half by the lower apexes, and one-half, by means of vertical tie-rods, by the upper apexes, distributing the load on the top and bottom chords to equal parts.

The length $= 259$ feet;

the number of triangles, 27;

$$\text{the depth} = \frac{18,5}{2} \text{ tang } 60° = 9,25 \times 1,73 = 16 \text{ feet.}$$

For a more simple calculation the length of 9,25, or one-half of the side of the triangle, may be called 1, or unit; then

The side of the triangle $= 2$;

the height $= 1,73$;

the whole length of girder $= 28$.

134.] The load and weight being each 5 tons for every connecting point, the position of the acting forces on the girder will be seen in Fig. 134.

For the calculation of horizontal strains, x and z, in the upper and lower chords, we have for the section, AMN, with M at one time, and N at another, as the point of rotation, and the value of

$$D = 10\left(\tfrac{1}{28} + \tfrac{2}{28} + \ldots + \tfrac{27}{28}\right) = 135 \text{ tons.}$$

The value of $D = 135$ tons, introduced.

135.] $0 = x_4 . 1,73 + 135 \times 7 - 10\,(1 + 2 + 3 + \ldots + 6)$; (Fig. 135.)

$$0 = -z_4 . 1,73 + 135 \times 8 - 10\,(1 + 2 + 3 + \ldots + 7).$$

These equations give the result,

$$x_4 = -425 \text{ tons;}$$

$$z_4 = +462 \quad \text{"}$$

In the same way for the other strains in x and z.

$$0 = x_1 . 1,73 + 135 \times 1;$$

fig.128

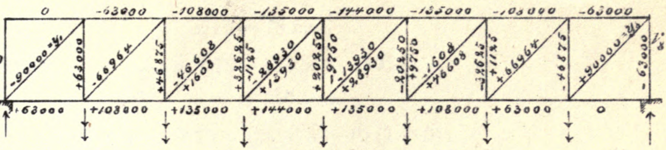


fig.129

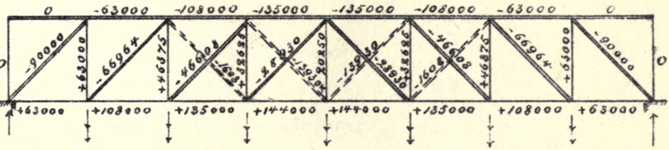


fig.130

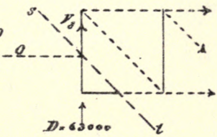


fig.131

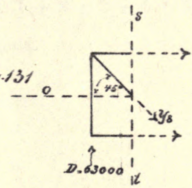
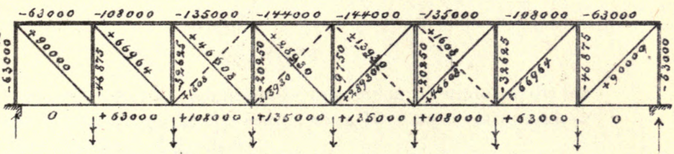


fig.132



$$x_1 = -78 \text{ tons.}$$

$$0 = -z_1 . 1{,}73 + 135 \times 2 - 10 . 1 ;$$

$$z_1 = +150 \text{ tons.}$$

$$0 = x_2 . 1{,}73 + 135 \times 3 - 10 (1 + 2) ;$$

$$x_2 = -216 \text{ tons.}$$

$$0 = -z_2 . 1{,}73 + 135 \times 4 - 10 (1 + 2 + 3) ;$$

$$z_2 = +277 \text{ tons};$$

$$0 = x_3 . 1{,}73 + 135 \times 5 - 10 (1 + 2 + 3 + 4) ;$$

$$x_3 = -338 \text{ tons};$$

$$0 = -z_3 . 1{,}73 + 135 \times 6 - 10 (1 + 2 + 3 + 4 + 5) ;$$

$$z_3 = +381 \text{ tons.}$$

Further, also,

$$x_5 = -494 \text{ tons};$$

$$z_5 = +520 \text{ “}$$

$$x_6 = -540 \text{ “}$$

$$z_6 = +555 \text{ “}$$

$$x_7 = -564 \text{ “}$$

$$z_7 = +566 \text{ “}$$

Remark.—For an approximate calculation of strain in the top and bottom flanges at the centre, when $10 \times 28 =$ entire load, or $140 =$ one-half of the load,

$$\frac{140 \times \frac{28}{4}}{1{,}73} = 566 \text{ tons.}$$

Calculation of Strains y and u in Diagonals.

The angle of diagonals with a horizontal line being 60°, for the strain in diagonals and tie-rods we have

$$y . \sin 60°, \quad \text{and } u . \sin 60°,$$

or

$$y . 0{,}866, \quad \text{and } u . 0{,}866.$$

136, 137. So for y_4 and u_4 in Figs. 136 and 137,

$$o = y_4 \,.\, 0{,}866 - D + 5 \times 6 + 5 \times 6;$$

$$o = - u_4 \,.\, 0{,}866 - D + 5 \times 7 + 5 \times 7.$$

In substituting the permanent and rolling load,

$$D = 5\left(\tfrac{1}{28} + \tfrac{2}{28} + \tfrac{3}{28} + \ldots + \tfrac{27}{28}\right) + 5\left(\tfrac{1}{28} + \tfrac{2}{28} + \tfrac{3}{28} \ldots + \tfrac{27}{28}\right);$$

and combining each time the share of each separate load of the pressure, D, with those directly-produced vertical strains, as in the Howe truss already shown, these equations will be

$$0 = y_4 \,.\, 0{,}866 - 5\left[\left(\tfrac{1}{28} + \tfrac{2}{28} + \ldots + \tfrac{21}{28}\right) - \left(1 - \tfrac{22}{28}\right) - \left(1 - \tfrac{23}{28}\right) - \ldots \left(1 - \tfrac{27}{28}\right)\right] - 5\left(\tfrac{1}{28} + \tfrac{2}{28} + \ldots + \tfrac{21}{28}\right)$$
$$+ 5\left[\left(1 - \tfrac{22}{28}\right) + \left(1 - \tfrac{23}{28}\right) + \ldots \left(1 - \tfrac{27}{28}\right)\right];$$

$$0 = - u_4 \,.\, 0{,}866 - 5\left[\left(\tfrac{1}{28} + \tfrac{2}{28} + \ldots + \tfrac{20}{28}\right) - \left(1 - \tfrac{21}{28}\right) - \left(1 - \tfrac{22}{28}\right) - \ldots \left(1 - \tfrac{27}{28}\right)\right] - 5\left(\tfrac{1}{28} + \tfrac{2}{28} + \ldots + \tfrac{20}{28}\right)$$
$$+ 5\left[\left(1 - \tfrac{21}{28}\right) + \left(1 - \tfrac{22}{28}\right) + \ldots \left(1 - \tfrac{27}{28}\right)\right];$$

and omitting in these equations at one time the positive (+) members and at another time the negative (—) members which were produced by the rolling load, we have

$$0 = y_4 \,.\, 0{,}866 - 5\left(\tfrac{1}{28} + \tfrac{2}{28} + \ldots + \tfrac{21}{28} - \tfrac{6}{28} - \tfrac{5}{28} - \tfrac{4}{28} - \ldots \tfrac{1}{28}\right) + 5\left(\tfrac{1}{28} + \tfrac{2}{28} + \ldots \tfrac{21}{28}\right);$$

$$y_4 = + 91 \text{ tons (max.)};$$

$$0 = y_4 \,.\, 0{,}866 - 5\left(\tfrac{1}{28} + \tfrac{2}{28} + \ldots + \tfrac{21}{28} - \tfrac{6}{28} - \tfrac{5}{28} - \ldots \tfrac{1}{28}\right) + 5\left(\tfrac{6}{28} + \tfrac{5}{28} + \ldots \tfrac{1}{28}\right);$$

$$y_4 = + 39 \text{ tons.}$$

$$0 = - u_4 \,.\, 0{,}866 - 5\left(\tfrac{1}{28} + \tfrac{2}{28} + \ldots \tfrac{20}{28} - \tfrac{7}{28} - \tfrac{6}{28} - \ldots \tfrac{1}{28}\right) + 5\left(\tfrac{7}{28} + \tfrac{6}{28} + \ldots \tfrac{1}{28}\right);$$

$$u_4 = - 32 \text{ tons.}$$

$$0 = - u_4 \,.\, 0{,}866 - 5\left(\tfrac{1}{28} + \tfrac{2}{28} + \ldots \tfrac{20}{28} - \tfrac{7}{28} - \tfrac{6}{28} - \ldots \tfrac{1}{28}\right) - 5\left(\tfrac{1}{28} + \tfrac{2}{28} + \ldots \tfrac{20}{28}\right);$$

$$u_4 = - 81 \text{ tons (min.)}.$$

In the same manner as for y_4 and u_4 the equations for the other diagonals are,

fig. 133.

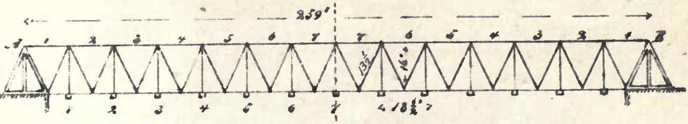


fig. 134

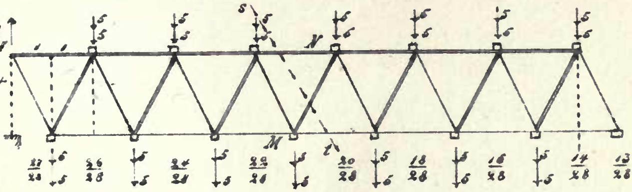


fig. 135

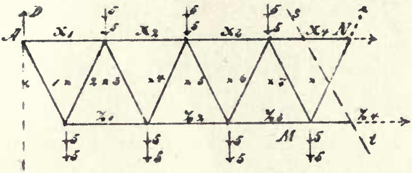


fig. 136

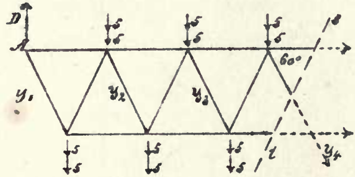


fig. 137

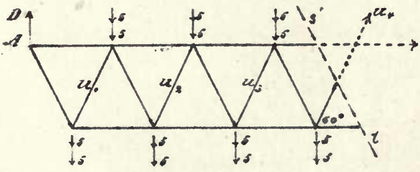


fig. 138

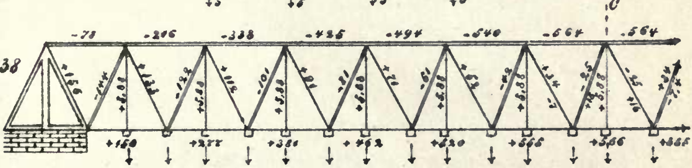
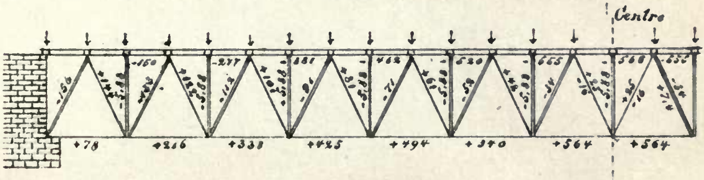


fig. 139



$$0 = y_1 . 0,866 - 5\left(\tfrac{1}{28} + \tfrac{2}{28} + \ldots \tfrac{27}{28}\right) - 5\left(\tfrac{1}{28} + \tfrac{2}{28} + \ldots \tfrac{27}{28}\right);$$

$$y_1 = +156 \text{ tons}, \qquad \text{and } y_1 = +78 \text{ tons}.$$

$$0 = -u_1 . 0,866 - 5\left(\tfrac{1}{28} + \tfrac{2}{28} + \ldots \tfrac{26}{28} - \tfrac{1}{28}\right) - 5\left(\tfrac{1}{28} + \tfrac{2}{28} + \ldots \tfrac{26}{28}\right) + 5\left(\tfrac{1}{28}\right);$$

$$u_1 = -72 \text{ tons}, \qquad \text{and } u_1 = -144 \text{ tons}.$$

$$0 = y_2 . 0,866 - 5\left(\tfrac{1}{28} + \tfrac{2}{28} + \ldots \tfrac{25}{28} - \tfrac{2}{28} - \tfrac{1}{28}\right) - 5\left(\tfrac{1}{28} + \tfrac{2}{28} + \ldots \tfrac{25}{28}\right) + 5\left(\tfrac{2}{28} + \tfrac{1}{28}\right);$$

$$y_2 = +133 \text{ tons}, \qquad \text{and } y_2 = +66 \text{ tons};$$

$$0 = -u_2 . 0,866 - 5\left(\tfrac{1}{28} + \tfrac{2}{28} + \ldots \tfrac{24}{28} - \tfrac{3}{28} - \tfrac{2}{28} - \tfrac{1}{28}\right) - 5\left(\tfrac{1}{28} + \tfrac{2}{28} + \ldots \tfrac{24}{28}\right) + 5\left(\tfrac{3}{28} + \tfrac{2}{28} + \tfrac{1}{28}\right);$$

$$u_2 = -59 \text{ tons}, \qquad \text{and } u_2 = -122 \text{ tons}.$$

Further,

$$y_3 = +112, \qquad \text{and } y_3 = +53;$$
$$u_3 = -46, \qquad \text{and } u_3 = -101;$$
$$y_5 = +71, \qquad \text{and } y_5 = +24;$$
$$u_5 = -17, \qquad \text{and } u_5 = -61;$$
$$y_6 = +52, \qquad \text{and } y_6 = +9;$$
$$u_6 = -0,8, \qquad \text{and } u_6 = -42;$$
$$y_7 = +34, \qquad \text{and } y_7 = -7,4;$$
$$u_7 = +16, \qquad \text{and } u_7 = -25.$$

For the other half of the truss the numbers are the same.

The suspenders bear, besides 5 tons moving load, a part of the permanent load; so that, when the weight of track = 24,75 tons, there is for each rod $\frac{24,75}{28} = 0,88$ tons more, or 5,88 tons.

138.] The results of the above calculations in skeleton 138.

139.] When the + and — signs in this skeleton are changed, it represents a truss for a deck-bridge. The suspenders in this case will be studs.

2. ISOMETRICAL TRUSS.

Plate 23, Fig. 140.] Calculation of a 143-feet span through bridge. (Fig. 140.)

p = permanent load per panel = 5,5 tons for two ribs.
q = rolling " " = 17,5 " " "
Together = 23,0 tons.

Depth of truss = 22,52 feet;
length of diagonals = 26 feet;
length of panels = 13 feet.

The weight and load of bridge acting upon the system, being
141.] $23 \times 10 = 230$ tons, makes for two ribs, but one single system $= \frac{230}{2} = 115$ tons, and it will be similar in Fig. 141.

$$D_1 = (p+q)\left(\tfrac{1}{11} + \tfrac{3}{11} + \tfrac{5}{11} + \tfrac{7}{11} + \tfrac{9}{11}\right) + \frac{p+q}{2} \cdot \tfrac{11}{11},$$

or $D = (p+q)\left(\tfrac{1}{11} + \tfrac{3}{11} + \tfrac{5}{11} + \tfrac{7}{11} + \tfrac{9}{11}\right) = 52{,}27$ tons on the left, and $D = 62{,}73$ tons on the right support.

On account of the symmetry of the systems (one left = the other right), we need only to make a calculation both for diagonals and flanges for one system, adding for the strain in flanges the covering parts of the two combined diagrams. (See Fig. 146.)

a. Calculation of Diagonals.

141.] For the diagonals we have again, as in the foregoing examples (see Howe Truss, Fig. 118), as example for y_4, Fig. 141,

$$0 = -y_4 \cdot \sin 60° - D + (p+q) + (p+q) + (p+q) \text{ (rot. } r \,.\, o);$$

$$0 = -y_4 \cdot 0{,}866 - (p+q)\left[\tfrac{1}{11} + \tfrac{3}{11} + \left(\tfrac{5}{11} - 1\right) + \left(\tfrac{7}{11} - 1\right) + \tfrac{9}{11} - 1)\right],$$

or $0 = -y_4 \cdot 0{,}866 - (p+q)\left[\tfrac{1}{11} + \tfrac{3}{11} - \left(1 - \tfrac{5}{11}\right) - \left(1 - \tfrac{7}{11}\right) - \left(1 - \tfrac{9}{11}\right)\right];$

$$0 = -y_4 \cdot 0{,}866 - (p+q)\left(\tfrac{1}{11} + \tfrac{3}{11} - \tfrac{6}{11} - \tfrac{4}{11} - \tfrac{2}{11}\right);$$

$$0 = -y_4 \cdot 0{,}866 + p \cdot \tfrac{8}{11} - q \cdot \tfrac{4}{11} + q \cdot \tfrac{12}{11}.$$

1.) $0 = -y_4 \cdot 0{,}866 + p \cdot \tfrac{8}{11} - q \cdot \tfrac{4}{11};$

2.) $0 = -y_4 \cdot 0{,}866 + p \cdot \tfrac{8}{11} + q \cdot \tfrac{12}{11}.$

I. $y_4 = -2{,}72$ tons (max. compr.);

II. $y_4 = +26{,}6$ " (max. tens.);

and according to the theorem in Sect. II., Fig. 80, the strains in diagonals joining at an unloaded point are of the same numerical amount, but of opposite character.

I. $u_4 = +\ 2{,}72$ tons (max. tens.);

II. $u_4 = +\ 26{,}6$ " (max. compr.).

In the same way,

$$0 = -\ y_1 \cdot 0{,}866 - D;$$

$$y_1 = -\frac{52{,}25}{0{,}866} = -\ 60;$$

$$u_1 = +\ 60;$$

$$0 = -\ y_2 \cdot 0{,}866 - (p+q)\left[\tfrac{1}{11} + \tfrac{3}{11} + \tfrac{5}{11} + \tfrac{7}{11} + (\tfrac{9}{11} - 1)\right];$$

$$0 = -\ y_2 \cdot 0{,}866 - p \cdot \tfrac{14}{11} - q \cdot \tfrac{16}{11} + q \cdot \tfrac{2}{11};$$

$$y_2 = -\ 37{,}4, \qquad \text{and } y_2 = -\ 4{,}41;$$

$$u_2 = +\ 37{,}4, \qquad \text{and } u_2 = +\ 4{,}41;$$

$$0 = -\ y_3 \cdot 0{,}866 - (p+q)\left[\tfrac{1}{11} + \tfrac{3}{11} + \tfrac{5}{11} + (\tfrac{7}{11} - 1) + (\tfrac{9}{11} - 1)\right];$$

$$0 = -\ y_3 \cdot 0{,}866 - p \cdot \tfrac{3}{11} - q \cdot \tfrac{9}{11} + q \cdot \tfrac{6}{11};$$

$$y_3 = -\ 18{,}3, \qquad \text{and } y_3 = +\ 10;$$

$$u_3 = +\ 18{,}3, \qquad \text{and } u_3 = -\ 10;$$

$$0 = -\ y_5 \cdot 0{,}866 - (p+q)\left[\tfrac{1}{11} + (\tfrac{3}{11} - 1) + (\tfrac{5}{11} - 1) + (\tfrac{7}{11} - 1) + (\tfrac{9}{11} - 1)\right];$$

$$0 = -\ y_5 \cdot 0{,}866 + p \cdot \tfrac{19}{11} - q \cdot \tfrac{1}{11} + q \cdot \tfrac{20}{11};$$

$$y_5 = +\ 47{,}4, \qquad \text{and } y_5 = +\ 9{,}2;$$

$$u_5 = -\ 47{,}4, \qquad \text{and } u_5 = -\ 9{,}2;$$

$$0 = -\ y_6 \cdot 0{,}866 - (p+q)\left[(\tfrac{1}{11} - 1) + (\tfrac{3}{11} - 1) + (\tfrac{5}{11} - 1) + (\tfrac{7}{11} - 1) + (\tfrac{9}{11} - 1)\right];$$

$$0 = -\ y_6 \cdot 0{,}866 - (p+q)\left(-\tfrac{10}{11} - \tfrac{8}{11} - \tfrac{6}{11} - \tfrac{4}{11} - \tfrac{2}{11}\right);$$

$$y_6 = +\ 72{,}1 \text{ tons.}$$

142.] The results for the combined system in skeleton 142.

Remark.—For the compression in the vertical posts *o*, *a* and 11 . *m* on the abutments, we have

$$0{,}866 \times 72{,}1 = 62{,}5 \text{ tons.}$$

And for an approximate estimate of strain at the centre of chords,

$23 \times 11 = 253$ the entire weight and load of bridge;

$126{,}5 =$ one-half the weight and load;

$$\frac{126{,}5 \times \frac{143}{4}}{22{,}52} = 200 \text{ tons.}$$

b. Calculation of Top and Bottom Chords (Flanges).

143.] $0 = x_1 \,.\, 22{,}52 + D \,.\, o$ (rot. $r \,.\, a$, Fig. 143);

$$x_1 = 0;$$

$$0 = - z_1 \,.\, 22{,}52 + D \,.\, 13 \text{ (rot. } r \,.\, 1\text{, Fig. 143);}$$

$$z_1 = \frac{52{,}27 \times 13}{22{,}52} = + 30{,}1;$$

$$0 = x_2 \,.\, 22{,}52 + D \,.\, 26 \text{ (rot. } r \,.\, c\text{, Fig. 143);}$$

$$x_2 = - \frac{52{,}27 \times 26}{22{,}52} = - 60{,}3;$$

$$0 = - z_2 \,.\, 22{,}52 + D \,.\, 39 - 23 \times 13 \text{ (rot. } r \,.\, 3\text{);}$$

$$z_2 = + \frac{52{,}27 \times 39 - 23 \times 13}{22{,}52} = + 77{,}2;$$

144.] $0 = x_3 \,.\, 22{,}52 + D \,.\, 52 - 23 \times 26$ (rot. $r \,.\, e$, Fig. 144);

$$x_3 = - \frac{52{,}27 \times 52 + 23 \times 26}{22{,}52} = - 94{,}1;$$

145.] $0 = - z_3 \,.\, 22{,}52 + D \,.\, 65 - 23\,(13 + 39)$ (rot. $r \,.\, 5$, Fig. 145);

$$z_3 = \frac{52{,}27 \times 65 - 23 \times 52}{22{,}52} = + 97{,}8;$$

$$0 = x_4 \,.\, 22{,}52 + D \,.\, 78 - 23\,(26 + 52) \text{ (rot. } r \,.\, g\text{);}$$

$$x_4 = - \frac{52{,}27 \times 78 + 23 \times 78}{22{,}52} = - 101{,}3;$$

$$0 = - z_4 \,.\, 22{,}52 + D \,.\, 91 - 23\,(13 + 39 + 65) \text{ (rot. } r \,.\, 7\text{);}$$

$$z_4 = \frac{52{,}27 \times 91 - 23 \times 117}{22{,}52} = + 91{,}7;$$

fig. 140

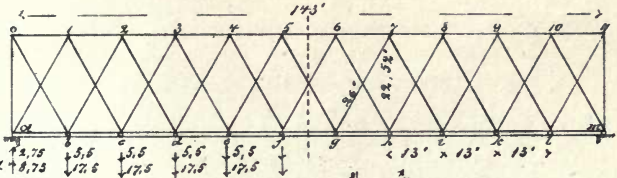


fig. 141.

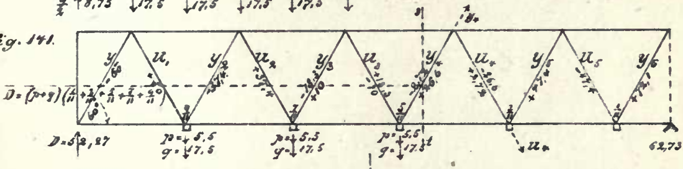


fig. 142

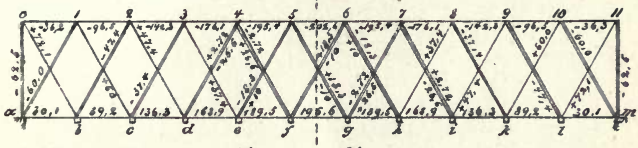


fig. 143

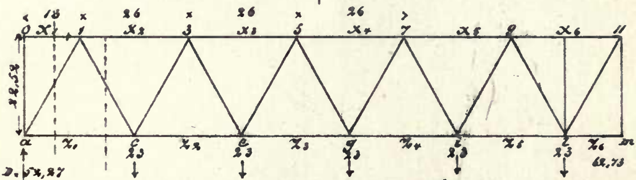


fig. 144

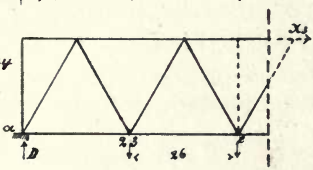


fig. 145

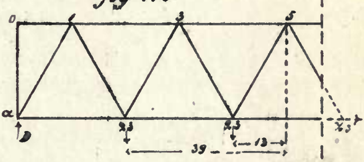


fig. 146

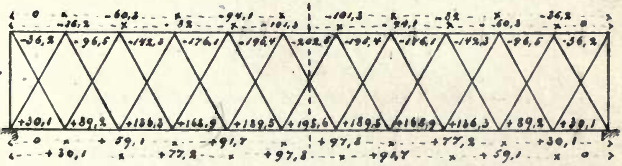
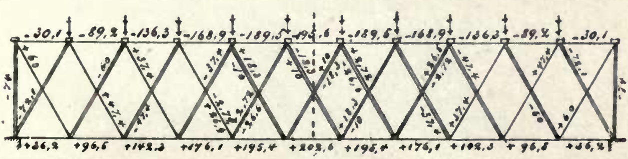


fig. 147



$$0 = x_5 \,.\, 22{,}52 + D \,.\, 104 - 23\,(26 + 52 + 78) \text{ (rot. } r\,.\,i);$$

$$x_5 = -\frac{52{,}27 \times 104 + 23 \times 156}{22{,}52} = -82{,}0\,;$$

$$0 = -z_5 \,.\, 22{,}52 + D \,.\, 117 - 23\,(13 + 39 + 65 + 91);$$

$$z_5 = \frac{52{,}27 \times 117 - 23 \times 208}{22{,}52} = +59{,}1\,;$$

$$0 = x_6 \,.\, 22{,}52 + D \,.\, 130 - 23\,(26 + 52 + 78 + 104)$$

$$x_6 = -\frac{52{,}27 \times 130 + 23 \times 260}{22{,}52} = -36{,}2\,;$$

$$0 = -z_6 \,.\, 22{,}52 + D \,.\, 143 - 23\,(13 + 39 + 65 + 91 + 117)\,;$$

$$z_6 = \frac{52{,}27 \times 143 - 23 \times 325}{22{,}52} = 0.$$

146.] The results of strain in the top and bottom flanges by the addition of strain for each single system are combined in Fig. 146.

147.] For a deck-bridge the strains in diagonals and flanges result from Fig. 147.

B. CAMBER IN TRUSSES, WITH PARALLEL TOP AND BOTTOM CHORDS.

After the definition of the resulting strains, it is a matter of importance to prevent deflection, which should be provided for in "laying out" the truss.

In the suspension truss (Fig. 10) no special attention to certain camber is required, each pair of tie-rods with combined vertical post forming an independent system, regulated by tie-bolts at the foot of the post.

Plate 24, Fig. 148.] In other trusses the difference in the length of diagonals and verticals would form arch-shaped chords, as demonstrated by Fig. 148[a, b].

This arrangement having some difficulties on account of the difference in the length of braces, another and better plan is fre-

quently adopted in making the division of panels in the bottom chord smaller than in the top chord.

149[a].] In this case the vertical connections will lose their parallel direction and intersect at a certain point in the vertical centre line. (Fig. 149.)

Approximately, this can be done by experience, though not sufficiently, in case a certain camber is prescribed.

For a correct calculation the annexed tables, showing the length of arches for degrees, minutes and seconds, will be useful. (See Tables, p. 77, *et seq.*)

Example.—(Truss, Fig. 140.)

Distance of the distinguished points $AB = 143$ feet;

depth of truss $= 22{,}52$ feet;

number of panels $= 11$.

First we find the radius in Fig. 149[a], according to Sect. I., Fig. B,

The distance AB being $2 \times 71{,}5 = 143$ feet;

the distance $CI = \dfrac{AD^2}{CD} + CD,$

or $$CI = \frac{71{,}5^2}{0{,}33333} + 0{,}33333;$$

$$\text{Diameter } CI = 15336{,}09 + 0{,}33333 = 15336{,}423;$$

$$\text{“} \quad EH = 15336{,}423 + 2 \times 22{,}52 = 15381{,}463;$$

or, also, $$\text{radius } AM = 7668{,}211 \text{ feet};$$

$$\text{“} \quad FM = 7690{,}731 \text{ “}$$

By means of the radii for a small camber ($\frac{1}{500}$ of the length), in general the difference of length of the top and bottom flanges may be derived approximately by the difference of the chords AB and FG, viz.:

$$\frac{Fd}{AD} = \frac{FM}{AM}, \qquad \text{or } Fd = AD \cdot \frac{FM}{AM};$$

$$Fd = 71{,}5 \times \frac{7690{,}731}{7668{,}211};$$

$$Fd = 71{,}709;$$

i. e., $$FG = 143{,}418;$$

and when $AB = 143{,}000$ subtracted,

the difference of chords = 0,418 ft. = 5,016 in.,

which, when divided by 11 (*i. e.*, by the number of panels),

$$\frac{5{,}016}{11} = 0{,}456 \text{ in.};$$

therefore, for a camber of 4 inches, the top flanges ought to be made 0,456 inches longer than the bottom flanges; the latter being 13 feet, the top flanges ought to be 13 feet and $\frac{7}{16}$ inches.

For 4 inches camber, a truss of 100 feet length, $9\frac{1}{2}$ feet depth, and 9 panels, the top flanges ought to be 0,339 inches longer than the bottom flanges.

Also, a truss with 14 panels, $9\frac{1}{2}$ feet depth, $5\frac{1}{2}$ feet in bottom flanges, and 5 feet $6\frac{1}{4}$ inches in top flanges, will have a camber of nearly 6 inches.

A truss of 27 panels, 4 feet depth, $2\frac{1}{2}$ feet in bottom flanges, and $\frac{1}{48}$ more in top flanges, will give a camber of nearly 2 inches.

In the preceding calculation the definition of the angle a at the centre, M, is avoided, otherwise, by geometrical rule, the arch is in relation to the whole circle as its angle at the centre is to 360°;

i. e., $$\frac{\text{arc}}{2.r.\pi} = \frac{a}{360^\circ}. \quad \text{or} \quad \frac{\text{arc}}{\text{diam.} \times 3{,}14} = \frac{a}{360^\circ}.$$

Now, when not the bottom chord, AB, but for a small camber the arch ACB is accepted to 143 feet, and the defined diameter of the preceding is retained, being in reality smaller, then

$$\frac{143}{15336{,}423 \times 3{,}141} = \frac{a}{360},$$

or $$a^\circ = \frac{360 \times 143}{15336{,}423 \times 3{,}141} = 1{,}068^\circ = 1^\circ\, 4{,}08';$$

therefore for the arch $ACB = x$,

$$\frac{x}{15336{,}423 \times 3{,}141} = \frac{1{,}068}{360},$$

or $$x = 142{,}909,$$

and for the arch $FEG = y$,

$$\frac{y}{15381{,}463 \times 3{,}141} = \frac{1{,}068}{360},$$

or $$y = 143{,}329;$$

$y - x = 143{,}329 - 142{,}909 = 0{,}420$ feet $= 5{,}04$ inches, the difference in the arches, which, divided by 11, (*i. e.*, by the number of panels),

$\frac{5{,}04}{11} = 0{,}458$ inches, the difference in the top and bottom flanges, as before.

More accurate still is the following calculation:

$$\frac{AD}{AM} = \sin\frac{a}{2} = \frac{71{,}5}{7668{,}211}\ 0{,}009342,$$

which gives by tables of nat. sin., cos., etc.,

$$\frac{a}{2} = 32{,}117' \text{ (approx.)}, \quad \text{or } a = 1^\circ\, 4{,}234' = 1^\circ\, 4'\, 14''.$$

The annexed tables show

For 1°	the figure	0,017453
" 4′	"	0,001164
" 14″	"	0,000068
		0,018685

which, by multiplication with the radius, $AM = r$, gives arc, $ACB = 0{,}018685 \times 7668{,}211 = 143{,}2805$ feet, and with the radius, $FM = R$,

$$\text{Arc } FEG = 0{,}018685 \times 7690{,}731 = 143{,}7013 \text{ feet.}$$

The difference of arches, therefore,

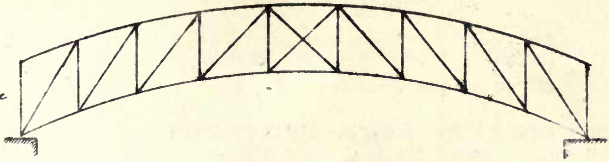
$$143{,}7013 - 143{,}2805 = 0{,}4208 \text{ feet.}$$

This result divided by the number of panels = 11.

$$\frac{0{,}4208}{11} = 0{,}038 \text{ feet, or } 0{,}456 \text{ inches.}$$

Thus $\frac{143{,}2805}{11} = 13{,}025$ feet (the bottom flanges),

and $13{,}025 + 0{,}038 = 13{,}063$ feet (the top flanges).



fig·148a.

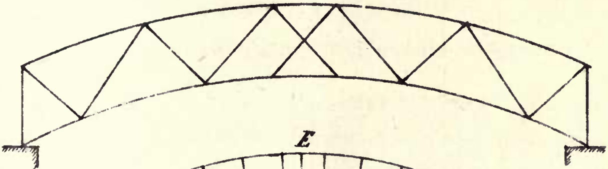
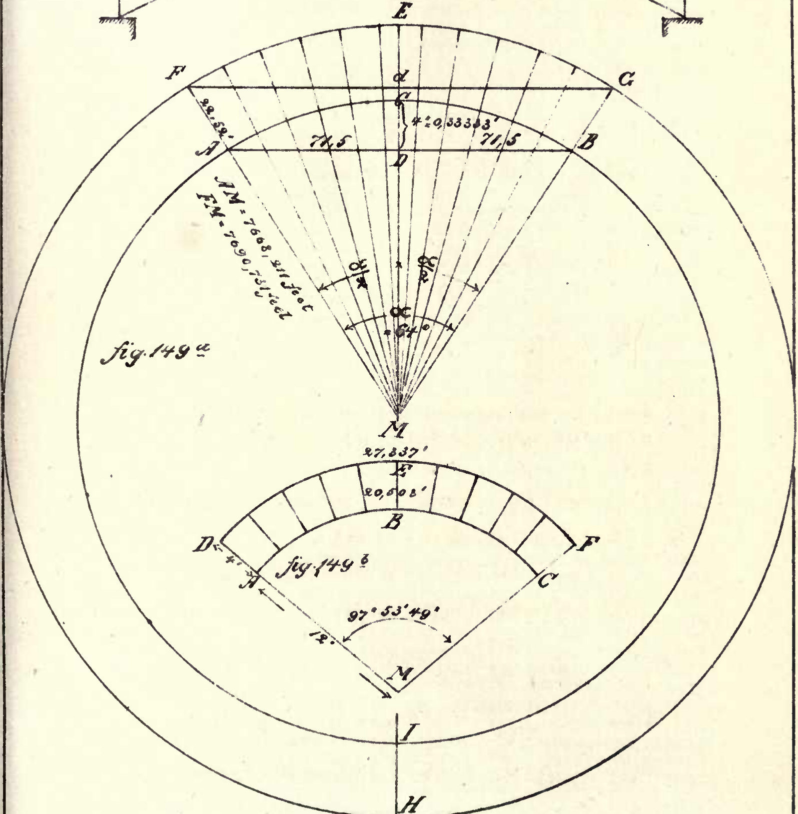


fig 148b.



fig·149a.

fig 149b.

TABLES CONTAINING THE LENGTH OF ARCHES FOR DEGREES, MINUTES AND SECONDS, FOR A RADIUS AS UNIT.

LENGTH OF ARCHES FOR EVERY DEGREE.

1	0.017453	46	0.802852	91	1.588250	136	2.373648	181	3.159046
2	0.034907	47	0.820305	92	1.605703	137	2.391101	182	3.176499
3	0.052360	48	0.837758	93	1.623156	138	2.408554	183	3.193953
4	0.069813	49	0.855211	94	1.640610	139	2.426008	184	3.211406
5	0.087267	50	0.872665	95	1.658063	140	2.443461	185	3.228859
6	0.104720	51	0.890118	96	1.675516	141	2.460914	186	3.246312
7	0.122173	52	0.907571	97	1.692969	142	2.478368	187	3.263766
8	0.139626	53	0.925025	98	1.710423	143	2.495821	188	3.281219
9	0.157080	54	0.942478	99	1.727876	144	2.513274	189	3.298672
10	0.174533	55	0.959931	100	1.745329	145	2.530727	190	3.316126
11	0.191986	56	0.977384	101	1.762783	146	2.548181	191	3.333579
12	0.209440	57	0.994838	102	1.780236	147	2.565634	192	3.351032
13	0.226893	58	1.012291	103	1.797689	148	2.583087	193	3.368486
14	0.244346	59	1.029744	104	1.815142	149	2.600541	194	3.385939
15	0.261799	60	1.047198	105	1.832596	150	2.617994	195	3.403392
16	0.279253	61	1.064651	106	1.850049	151	2.635447	196	3.420845
17	0.296706	62	1.082104	107	1.867502	152	2.652901	197	3.438299
18	0.314159	63	1.099557	108	1.884956	153	2.670354	198	3.455752
19	0.331613	64	1.117011	109	1.902409	154	2.687807	199	3.473205
20	0.349066	65	1.134464	110	1.919862	155	2.705260	200	3.490659
21	0.366519	66	1.151917	111	1.937316	156	2.722714	201	3.508112
22	0.383972	67	1.169371	112	1.954769	157	2.740167	202	3.525565
23	0.401426	68	1.186824	113	1.972222	158	2.757620	203	3.543018
24	0.418879	69	1.204277	114	1.989675	159	2.775074	204	3.560472
25	0.436332	70	1.221731	115	2.007129	160	2.792527	205	3.577925
26	0.453786	71	1.239184	116	2.024582	161	2.809980	206	3.595378
27	0.471239	72	1.256637	117	2.042035	162	2.827433	207	3.612832
28	0.488692	73	1.274090	118	2.059489	163	2.844887	208	3.630285
29	0.506145	74	1.291544	119	2.076942	164	2.862340	209	3.647738
30	0.523599	75	1.308997	120	2.094395	165	2.879793	210	3.665191
31	0.541052	76	1.326450	121	2.111848	166	2.897247	211	3.682645
32	0.558505	77	1.343904	122	2.129302	167	2.914700	212	3.700098
33	0.575959	78	1.361357	123	2.146755	168	2.932153	213	3.717551
34	0.593412	79	1.378810	124	2.164208	169	2.949606	214	3.735005
35	0.610865	80	1.396263	125	2.181662	170	2.967060	215	3.752458
36	0.628319	81	1.413717	126	2.199115	171	2.984513	216	3.769911
37	0.645772	82	1.431170	127	2.216568	172	3.001966	217	3.787364
38	0.663225	83	1.448623	128	2.234021	173	3.019420	218	3.804818
39	0.680678	84	1.466077	129	2.251475	174	3.038673	219	3.822271
40	0.698132	85	1.483530	130	2.268928	175	3 054326	220	3.839724
41	0.715585	86	1.500983	131	2.286381	176	3.071780	221	3.857178
42	0.733038	87	1.518436	132	2.303835	177	3.089233	222	3.874631
43	0.750492	88	1.535890	133	2.321288	178	3.106686	223	3.892084
44	0.767945	89	1.553343	134	2.338741	179	3.124139	224	3.909538
45	0.785398	90	1.570796	135	2.356195	180	3.141592	225	3.926991

LENGTH OF ARCHES FOR EVERY DEGREE. (*Continued.*)

226	3.944444	261	4.555309	296	5.166175	331	5.777040
227	3.961897	262	4.572763	297	5.183628	332	5.794493
228	3.979351	263	4.590216	298	5.201081	333	5.811946
229	3.996804	264	4.607669	299	5.218535	334	5.829400
230	4.014257	265	4.625123	300	5.235988	335	5.846853
231	4.031711	266	4.642576	301	5.253441	336	5.864306
232	4.049164	267	4.660029	302	5.270894	337	5.881760
233	4.066617	268	4.677482	303	5.288348	338	5.899213
234	4.084070	269	4.694936	304	5.305801	339	5.916666
235	4.101524	270	4.712389	305	5.323254	340	5.934120
236	4.118977	271	4.729842	306	5.340708	341	5.951573
237	4.136430	272	4.747296	307	5.358161	342	5.969026
238	4.153884	273	4.764749	308	5.375614	343	5.986479
239	4.171337	274	4.782202	309	5.393067	344	6.003933
240	4.188790	275	4.799655	310	5.410521	345	6.021386
241	4.206244	276	4.817109	311	5.427974	346	6.038839
242	4.223697	277	4.834562	312	5.445427	347	6.056293
243	4.241150	278	4.852015	313	5.462881	348	6.073746
244	4.258603	279	4.869469	314	5.480334	349	6.091200
245	4.276057	280	4.886922	315	5.497787	350	6.108652
246	4.293510	281	4.904375	316	5.515240	351	6.126106
247	4.310963	282	4.921829	317	5.532694	352	6.143559
248	4.328417	283	4.939282	318	5.550147	353	6.161012
249	4.345870	284	4.956735	319	5.567600	354	6.178466
250	4.363323	285	4.974188	320	5.585054	355	6.195919
251	4.380776	286	4.991642	321	5.602507	356	6.213372
252	4.398230	287	5.009095	322	5.619960	357	6.230825
253	4.415683	288	5.026548	323	5.637414	358	6.248279
254	4.433136	289	5.044002	324	5.654867	359	6.265732
255	4.450590	290	5.061455	325	5.672320	360	6.283185
256	4.468043	291	5.078908	326	5.689773		
257	4.485496	292	5.096361	327	5.707227		
258	4.502950	293	5.113815	328	5.724680		
259	4.520403	294	5.131268	329	5.742133		
260	4.537856	295	5.148721	330	5.759587		

LENGTH OF ARCHES FOR EVERY MINUTE.

1	0.000291	16	0.004654	31	0.009018	46	0.013381
2	0.000582	17	0.004945	32	0.009308	47	0.013672
3	0.000873	18	0.005236	33	0.009599	48	0.013963
4	0.001164	19	0.005527	34	0.009890	49	0.014254
5	0.001454	20	0.005818	35	0.010181	50	0.014544
6	0.001745	21	0.006109	36	0.010472	51	0.014835
7	0.002036	22	0.006400	37	0.010763	52	0.015126
8	0.002327	23	0.006690	38	0.011054	53	0.015417
9	0.002618	24	0.006981	39	0.011345	54	0.015708
10	0.002909	25	0.007272	40	0.011636	55	0.015999
11	0.003200	26	0.007563	41	0.011926	56	0.016290
12	0.003491	27	0.007854	42	0.012217	57	0.016581
13	0.003782	28	0.008145	43	0.012508	58	0.016872
14	0.004072	29	0.008436	44	0.012799	59	0.017162
15	0.004363	30	0.008727	45	0.013090	60	0.017453

LENGTH OF ARCHES FOR EVERY SECOND.

1	0.000005	16	0.000078	31	0.000150	46	0.000223
2	0.000010	17	0.000082	32	0.000155	47	0.000228
3	0.000015	18	0.000087	33	0.000160	48	0.000233
4	0.000020	19	0.000092	34	0.000165	49	0.000238
5	0.000024	20	0.000097	35	0.000170	50	0.000242
6	0.000030	21	0.000102	36	0.000175	51	0.000247
7	0.000034	22	0.000107	37	0.000179	52	0.000252
8	0.000039	23	0.000112	38	0.000184	53	0.000257
9	0.000044	24	0.000116	39	0.000189	54	0.000262
10	0.000049	25	0.000121	40	0.000194	55	0 000267
11	0.000053	26	0.000126	41	0.000199	56	0.000272
12	0.000058	27	0.000131	42	0.000204	57	0.000276
13	0.000063	28	0.000136	43	0.000209	58	0.000281
14	0.000068	29	0.000141	44	0.000213	59	0.000286
15	0.000073	30	0.000145	45	0.000218	60	0.000291

Mode of Application.—When the angle, a, at the centre, M, of a part of a circle is known, then the radius must be multiplied with the sum of the figures attached to those degrees, minutes and seconds in the tables, and the length of the arch will be obtained.

149[b].] For example, we want to know the length of an arch for 97° 53′ 49″, when the radius of the circle is 12 feet. (Fig. 149[b].)

For a radius as unit, the arch for 97° = 1.692969
" " " " 53′ = 0.015417
" " " " 49″ = 0.000238
1.708624

This, multiplied by the radius, 12, gives 20.503488 feet, the length of the arch *ABC*, and when multiplied by the radius, 16, the length of arch = 27.337984.

Reversed, when the length of an arch = 98.765432 feet, and the radius of the circle = 20 feet, then for the angle at centre, M,

$$\frac{98.765432}{20} = 4.938272$$

In the table where an arch = 282° = 4.921829
Remainder, 0.016443
" " " = 56′ = 0.016290
Remainder, 0.000153
" " " = 31″ = 0.000150
Balance, 0.000003

therefore the angle = 282° 56′ 31″.

[Plates 22, 23 and 24—embracing Figs. 133 to 149.]

C. PARABOLIC GIRDER OF 48 FEET, OR 16 METER, SPAN.

(With Single Diagonal System.)

Weight of girder = 500 kilograms per meter, or 1000 kilograms on each apex.

Plate 25, Fig. 150. Rolling load = 2500 kilograms per meter, or 5000 kilograms on each apex. (Fig. 150.)

151.] The following skeleton (Fig. 151) shows the distribution of weight and load:

For the reactive force, D, of abutments, we have

$$D = 1000\left(\tfrac{1}{8}+\tfrac{2}{8}+\tfrac{3}{8}+\tfrac{4}{8}+\tfrac{5}{8}+\tfrac{6}{8}+\tfrac{7}{8}\right)+5000\left(\tfrac{1}{8}+\tfrac{2}{8}+\tfrac{3}{8}+\tfrac{4}{8}+\tfrac{5}{8}+\tfrac{6}{8}+\tfrac{7}{8}\right);$$

152.] For the strain in x_1 (Fig. 152) is

$$0 = x_1 \cdot \tfrac{7}{8} + D \cdot 2 \text{ (rot. } r \cdot C);$$

and when the value of D is substituted,

$$0 = x_1 \cdot \tfrac{7}{8} + 1000\left(\tfrac{1}{8}+\tfrac{2}{8}+\tfrac{3}{8}+\tfrac{4}{8}+\tfrac{5}{8}+\tfrac{6}{8}+\tfrac{7}{8}\right)2+5000\left(\tfrac{1}{8}+\tfrac{2}{8}+\tfrac{3}{8}+\tfrac{4}{8}+\tfrac{5}{8}+\tfrac{6}{8}+\tfrac{7}{8}\right)\cdot 2;$$

$$x_1 = -\ 48000 \text{ kil.};$$

and for z_1 from the same figure,

$$0 = -z_1 \cdot 0{,}8 + D \cdot 2 \text{ (rot. } r \cdot B);$$

$$0 = -z_1 \cdot 0{,}8 + 1000\left(\tfrac{1}{8}+\tfrac{2}{8}+\ldots+\tfrac{7}{8}\right)\cdot 2+5000\left(\tfrac{1}{8}+\tfrac{2}{8}+\ldots+\tfrac{7}{8}\right)\cdot 2;$$

$$z_1 = +\ 52500 \text{ kil.}$$

153.] For V we take as the point of rotation the intersection R of x_1 and z_2, and it is

$$0 = -V_1 \cdot 2{,}8 - D \cdot 0{,}8; \qquad \text{(Fig. 153.)}$$

$$0 = -V_1 \cdot 2{,}8 - 1000\left(\tfrac{1}{8}+\tfrac{2}{8}+\tfrac{3}{8}+\ldots+\tfrac{7}{8}\right)\cdot 0{,}8$$
$$-\ 5000\left(\tfrac{1}{8}+\tfrac{2}{8}+\tfrac{3}{8}+\ldots+\tfrac{7}{8}\right)\cdot 0{,}8;$$

$$V_1 = -\ 6000 \text{ kil.};$$

154.] For x_2 (rot. $r \cdot E$, Fig. 154) is

$$0 = x_2 \cdot 1{,}5 + D \cdot 4 - 1000 \times 2 - 5000 \times 2,$$

or $0 = x_2 . 1{,}5 + 1000 \left(\frac{1}{8} + \frac{2}{8} + \ldots \frac{7}{8}\right) . 4 + 5000 \left(\frac{1}{8} + \frac{2}{8} + \ldots \frac{7}{8}\right) . 4 - 1000 \times 2 - 5000 \times 2;$

and now, according to the rule formerly given,

$$0 = x_2 . 1{,}5 + 1000 \left[\left(\frac{1}{8} + \frac{2}{8} + \ldots \frac{6}{8}\right) . 4 + \left(\frac{7}{8} \times 4 - 2\right)\right];$$
$$+ 5000 \left[\left(\frac{1}{8} + \frac{2}{8} + \ldots \frac{6}{8}\right) . 4 + \left(\frac{7}{8} \times 4 - 2\right)\right];$$
$$x_2 = - 48000 \text{ kil.}$$

154.] For y_2 (rot. $r . R$, Fig. 154) is

$$0 = y_2 . 1{,}68 - 1000 \left(\frac{1}{8} + \frac{2}{8} + \ldots \frac{7}{8}\right) . 0{,}8 - 5000 \left(\frac{1}{8} + \frac{2}{8} + \ldots \frac{7}{8}\right) . 0{,}8 + 1000 \times 2{,}8 + 5000 \times 2{,}8;$$

$$0 = y_2 . 1{,}68 - 1000 \left[\left(\frac{1}{8} + \frac{2}{8} + \ldots \frac{6}{8}\right) . 0{,}8 - \left(2{,}8 - \frac{7}{8} \times 0{,}8\right)\right]$$
$$- 5000 \left(\frac{1}{8} + \frac{2}{8} + \ldots \frac{6}{8}\right) . 0{,}8 + 5000 \left(2{,}8 - \frac{7}{8} \times 0{,}8\right);$$

and omitting from the movable load at one time the members with the symbol +, and at another time with the symbol —,

$$0 = y_2 . 1{,}68 - 1000 \left[\left(\frac{1}{8} + \frac{2}{8} + \ldots \frac{6}{8}\right) . 0{,}8 - \left(2{,}8 - \frac{7}{8} \times 0{,}8\right)\right]$$
$$- 5000 \left(\frac{1}{8} + \frac{2}{8} + \ldots \frac{6}{8}\right) . 0{,}8,$$

or $$y_2 = + 6250 \text{ kil.},$$

and $$0 = y_2 . 1{,}68 - 1000 \left[\left(\frac{1}{8} + \frac{2}{8} + \ldots \frac{6}{8}\right) . 0{,}8 - \left(2{,}8 - \frac{7}{8} \times 0{,}8\right)\right]$$
$$+ 5000 \left(2{,}8 - \frac{7}{8} \times 0{,}8\right);$$
$$y_2 = - 6250 \text{ kil.}$$

In omitting no member of the first equation, y_2 will $= 0$, as per example for a full load. (See remarks on Parabolic Girders and the Arched Truss.)

For z_2 (rot. $r . B$) and the members in the prescribed form directly arranged, will be from Fig. 154.

$$0 = z_2 . 0{,}835 + 1000 \left(\frac{1}{8} + \frac{2}{8} + \ldots + \frac{7}{8}\right) 2 + 5000 \left(\frac{1}{8} + \frac{2}{8} + \ldots \frac{7}{8}\right) 2;$$
$$z_2 = + 50300 \text{ kil.}$$

155.] For V_2 (rot. $r . S$) we find from Fig. 155,

$$0 = - V_2 . 8 - 1000 \left[\left(\frac{1}{8} + \frac{2}{8} + \ldots \frac{6}{8}\right) . 4 - \left(6 - \frac{7}{8} \times 4\right)\right]$$
$$- 5000 \left(\frac{1}{8} + \frac{2}{8} + \ldots \frac{6}{8}\right) + 5000 \left(6 - \frac{7}{8} \times 4\right);$$

$$0 = -V_2 . 8 - 1000 \left[\left(\tfrac{1}{8} + \tfrac{2}{8} + \ldots \tfrac{6}{8}\right) . 4 - \left(6 - \tfrac{7}{8} \times 4\right) \right] - 5000 \left(\tfrac{1}{8} + \tfrac{2}{8} + \ldots \tfrac{6}{8}\right) . 4 ;$$

$$V_2 = -7560 \text{ kil.},$$

and $$0 = -V_2 . 8 - 1000 \left[\left(\tfrac{1}{8} + \tfrac{2}{8} + \ldots \tfrac{6}{8}\right) . 4 - \left(6 - \tfrac{7}{8} \times 4\right) \right] + 5000 \left(6 - \tfrac{7}{8} \times 4\right) ;$$

$$V_2 = +560 \text{ kil.}$$

In the same way we find for the remaining,

$$0 = x_3 . 1{,}875 + 1000 \left[\left(\tfrac{1}{8} + \tfrac{2}{8} + \ldots \tfrac{5}{8}\right) . 6 + \left(\tfrac{6}{8} \times 6 - 2\right) + \left(\tfrac{7}{8} \times 6 - 4\right) \right]$$
$$+ 5000 \left[\left(\tfrac{1}{8} + \tfrac{2}{8} + \ldots \tfrac{5}{8}\right) . 6 + \left(\tfrac{6}{8} \times 6 - 2\right) + \left(\tfrac{7}{8} \times 6 - 4\right) \right] ;$$

$$x_3 = -48000 \text{ kil.};$$

$$0 = y_3 . 5{,}47 - 1000 \left[\left(\tfrac{1}{8} + \tfrac{2}{8} + \ldots \tfrac{5}{8}\right) . 4 - \left(8 - \tfrac{6}{8} \times 4\right) - \left(6 - \tfrac{7}{8} \times 4\right) \right]$$
$$- 5000 \left(\tfrac{1}{8} + \tfrac{2}{8} + \ldots \tfrac{5}{8}\right) . 4 + 5000 \left(8 - \tfrac{6}{8} \times 4\right) + 5000 \left(6 - \tfrac{7}{8} \times 4\right) ;$$

$$y_3 = +6850 \text{ kil.},$$

and $$y_3 = -6850 \text{ ``}$$

$$0 = -z_3 . 1{,}474 + 1000 \left[\left(\tfrac{1}{8} + \ldots \tfrac{6}{8}\right) . 4 + \left(\tfrac{7}{8} \times 4 - 2\right) \right]$$
$$+ 5000 \left[\left(\tfrac{1}{8} + \ldots \tfrac{6}{8}\right) . 4 + \left(\tfrac{7}{8} \times 4 - 2\right) \right] ;$$

$$z_3 = +48900 \text{ kil};$$

$$0 = -V_3 . 30 - 1000 \left[\left(\tfrac{1}{8} + \ldots \tfrac{5}{8}\right) . 24 - \left(28 - \tfrac{6}{8} \times 24\right) - \left(26 - \tfrac{7}{8} \times 24\right) \right]$$
$$- 5000 \left(\tfrac{1}{8} + \ldots \tfrac{5}{8}\right) . 24 + 5000 \left(28 - \tfrac{6}{8} \times 24\right) + 5000 \left(26 - \tfrac{7}{8} \times 24\right) ;$$

$$V_3 = +1500 \text{ kil.},$$

and $$V_3 = -8500 \text{ ``}$$

$$0 = x_4 . 2 + 1000 \left[\left(\tfrac{1}{8} + \ldots \tfrac{4}{8}\right) . 8 + \left(\tfrac{5}{8} \times 8 - 2\right) + \left(\tfrac{6}{8} \times 8 - 4\right) + \left(\tfrac{7}{8} \times 8 - 6\right) \right]$$
$$+ 5000 \left[\left(\tfrac{1}{8} + \ldots \tfrac{4}{8}\right) . 8 + \left(\tfrac{5}{8} \times 8 - 2\right) + \tfrac{6}{8} \times 8 - 4\right) + \left(\tfrac{7}{8} \times 8 - 6\right) \right] ;$$

$$y_4 = -48000 \text{ kil.};$$

$$0 = y_4 . 21{,}2 - 1000 \left[\left(\tfrac{1}{8} + \ldots \tfrac{4}{8}\right) . 24 - \left(30 - \tfrac{5}{8} \times 24\right) - \left(28 - \tfrac{6}{8} \times 24\right) - \left(26 - \tfrac{7}{8} \times 24\right) \right]$$

$$- 5000 \left(\tfrac{1}{8} + \ldots \tfrac{4}{8}\right) . 24 + 5000 \left[\left(30 - \tfrac{5}{8} \times 24\right) + \left(28 - \tfrac{6}{8} \times 24\right) + \left(26 - \tfrac{7}{8} \times 24\right)\right];$$

$$y_4 = + 7080 \text{ kil.},$$

and $$y_4 = - 7080 \text{ “}$$

$$0 = - z_4 . 1{,}873 + 1000 \left[\left(\tfrac{1}{8} + \ldots \tfrac{5}{8}\right) . 6 + \left(\tfrac{6}{8} \times 6 - 2\right) + \left(\tfrac{7}{8} \times 6 - 4\right)\right]$$

$$+ 5000 \left[\left(\tfrac{1}{8} + \ldots \tfrac{5}{8}\right) . 6 + \left(\tfrac{6}{8} \times 6 - 2\right) + \left(\tfrac{7}{8} \times 6 - 4\right)\right];$$

$$z_4 = + 48100 \text{ kil.}$$

The equations now following are for the section of figure regarded to the right of the cut *st.*

$$0 = - V_4 . 32 + 1000 \left[\left(\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8}\right) 24 - \left(32 - \tfrac{4}{8} \times 24\right) - \left(30 - \tfrac{5}{8} \times 24\right) - \left(28 - \tfrac{6}{8} \times 24\right) - \left(26 - \tfrac{7}{8} \times 24\right)\right]$$

$$+ 5000 \left(\tfrac{1}{8} + \tfrac{2}{8} + \tfrac{3}{8}\right) 24 - 5000 \left[\left(32 - \tfrac{4}{8} \times 24\right) + \left(30 - \tfrac{5}{8} \times 24\right) + \left(28 - \tfrac{6}{8} \times 24\right) + \left(26 - \tfrac{7}{8} \times 24\right)\right];$$

$$V_4 = + 1800 \text{ kil.},$$

and $$V_4 = - 8800 \text{ “}$$

$$0 = - x_5 . 1{,}875 - 1000 \left[\left(\tfrac{1}{8} + \ldots \tfrac{5}{8}\right) 6 + \left(\tfrac{6}{8} \times 6 - 2\right) + \left(\tfrac{7}{8} \times 6 - 4\right)\right]$$

$$- 5000 \left[\left(\tfrac{1}{8} + \ldots \tfrac{5}{8}\right) 6 + \left(\tfrac{6}{8} \times 6 - 2\right) + \left(\tfrac{7}{8} \times 6 - 4\right)\right];$$

$$x_5 = - 48000 \text{ kil.};$$

$$0 = y_5 . 21{,}88 + 1000 \left[\left(\tfrac{1}{8} + \ldots \tfrac{4}{8}\right) 24 - \left(30 - \tfrac{5}{8} \times 24\right) - \left(28 - \tfrac{6}{8} \times 24\right) - 26 - \tfrac{7}{8} \times 24\right)\right]$$

$$+ 5000 \left(\tfrac{1}{8} + \ldots \tfrac{4}{8}\right) 24 - 5000 \left[\left(30 - \tfrac{5}{8} \times 24\right) + \left(28 - \tfrac{6}{8} \times 24\right) + \left(26 - \tfrac{7}{8} \times 24\right)\right];$$

$$y_5 = + 6850 \text{ kil.},$$

and $$y_5 = - 6850 \text{ “}$$

$$0 = z_5 . 1{,}996 - 1000 \left[\left(\tfrac{1}{8} + \ldots \tfrac{4}{8}\right) 8 + \left(\tfrac{5}{8} \times 8 - 2\right) + \left(\tfrac{6}{8} \times 8 - 4\right) + \left(\tfrac{7}{8} \times 8 - 6\right)\right]$$

$$- 5000 \left[\left(\tfrac{1}{8} + \ldots \tfrac{4}{8}\right) 8 + \left(\tfrac{5}{8} \times 8 - 2\right) + \left(\tfrac{6}{8} \times 8 - 4\right) + \left(\tfrac{7}{8} \times 8 - 6\right)\right];$$

$$z_5 = + 48100 \text{ kil.};$$

$$0 = - V_5 . 10 + 1000 [(\tfrac{1}{8} + \ldots \tfrac{4}{8}) 4 - (10 - \tfrac{5}{8} \times 4) - (8 - \tfrac{6}{8} \times 4) - (6 - \tfrac{7}{8} \times 4)]$$

$$+ 5000 [(\tfrac{1}{8} + \ldots \tfrac{4}{8}) 4 - 5000 [(10 - \tfrac{5}{8} \times 4) + (8 - \tfrac{6}{8} \times 4) + 6 - \tfrac{7}{8} \times 4)];$$

$$V_5 = + 1500 \text{ kil.},$$

and $$V_5 = - 8500 \text{ ``}$$

$$0 = - x_6 . 1{,}5 - 1000 [(\tfrac{1}{8} + \ldots \tfrac{6}{8}) . 4 + (\tfrac{7}{8} \times 4 - 2)] - 5000 [(\tfrac{1}{8} + \ldots \tfrac{6}{8} 4 + (\tfrac{7}{8} \times 4 - 2)];$$

$$x_6 = - 48000 \text{ kil.};$$

$$0 = y_6 . 6 + 1000 [(\tfrac{1}{8} + \ldots \tfrac{5}{8}) 4 - (8 - \tfrac{6}{8} \times 4) - (6 - \tfrac{7}{8} \times 4)]$$

$$+ 5000 (\tfrac{1}{8} + \ldots \tfrac{5}{8}) 4 - 5000 [(8 - \tfrac{6}{8} \times 4) + (6 - \tfrac{7}{8} \times 4)];$$

$$y_6 = + 6250 \text{ kil.},$$

and $$y_6 = - 6250 \text{ ``}$$

$$0 = z_6 . 1{,}84 - 1000 [(\tfrac{1}{8} + \ldots \tfrac{5}{8}) 6 + (\tfrac{6}{8} \times 6 - 2) + (\tfrac{7}{8} \times 6 - 4)]$$

$$- 5000 [(\tfrac{1}{8} + \ldots \tfrac{5}{8}) 6 + (\tfrac{6}{8} \times 6 - 2) + (\tfrac{7}{8} \times 6 - 4)];$$

$$z_6 = + 48900 \text{ kil.};$$

$$0 = - V_6 . 4{,}8 + 1000 [(\tfrac{1}{8} + \ldots \tfrac{5}{8}) 0{,}8 - (4{,}8 - \tfrac{6}{8} \times 0{,}8) - (2{,}8 - \tfrac{7}{8} \times 0{,}8)]$$

$$+ 5000 (\tfrac{1}{8} + \ldots \tfrac{5}{8}) 0{,}8 - 5000 [(4{,}8 - \tfrac{6}{8} \times 0{,}8) + (2{,}8 - \tfrac{7}{8} \times 0{,}8)];$$

$$V_6 = + 560 \text{ kil.},$$

and $$V_6 = - 7560 \text{ ``}$$

$$0 = - x_7 . 0{,}875 - 1000 (\tfrac{1}{8} + \ldots \tfrac{7}{8}) 2 - 5000 (\tfrac{1}{8} + \ldots \tfrac{7}{8}) 2;$$

$$x_7 = - 48000 \text{ kil};$$

$$0 = y_7 . 1{,}92 + 1000 [(\tfrac{1}{8} + \ldots \tfrac{6}{8}) 0{,}8 - (2{,}8 - \tfrac{7}{8} \times 0{,}8)]$$

$$+ 5000 (\tfrac{1}{8} + \ldots \tfrac{6}{8}) 0{,}8 - 5000 (2{,}8 - \tfrac{7}{8} \times 0{,}8);$$

$$y_7 = + 5470 \text{ kil.},$$

and $$y_7 = - 5470 \text{ ``}$$

$$0 = z_7 . 1{,}43 - 1000 [(\tfrac{1}{8} + \ldots \tfrac{6}{8}) 4 + (\tfrac{7}{8} . 4 - 2)] - 5000 [(\tfrac{1}{8} + \ldots \tfrac{6}{8}) 4 + (\tfrac{7}{8} \times 4 - 2)];$$

$$z_7 = + 50300 \text{ kil.};$$

fig 150

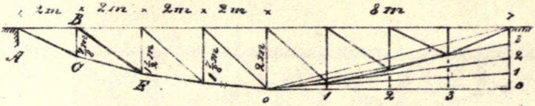


fig 151

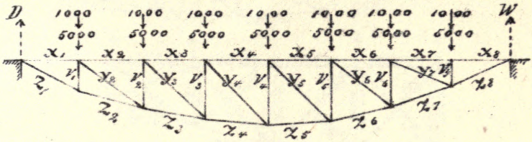


fig 152

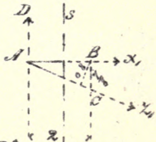


fig 153

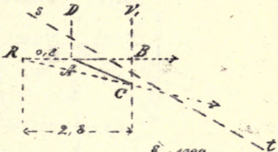


fig 154

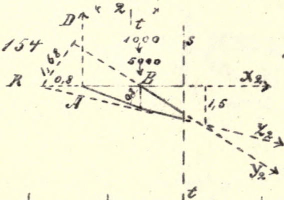
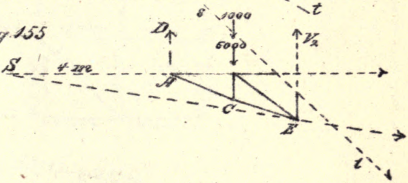


fig 155



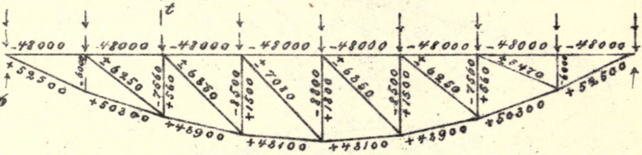


fig 156

fig 157

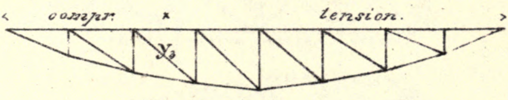


fig 158

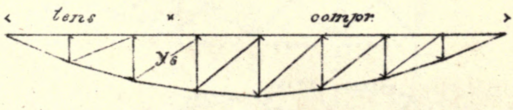


fig 159

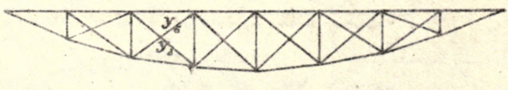


fig.160

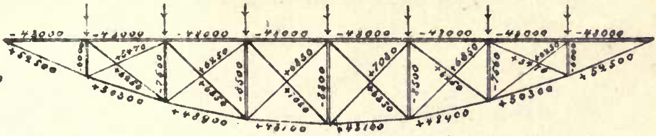


fig.161

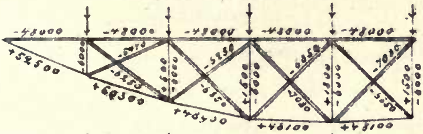


fig.162

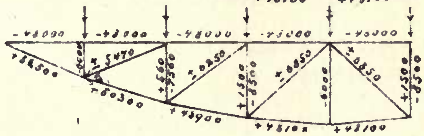


fig.163

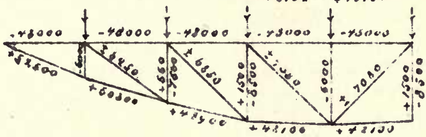


fig.164

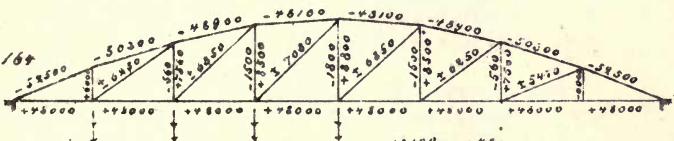


fig.165

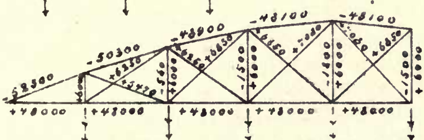


fig.166

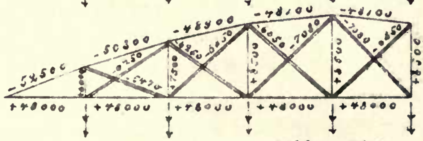


fig.167

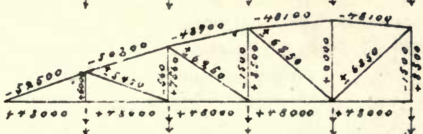


fig.168

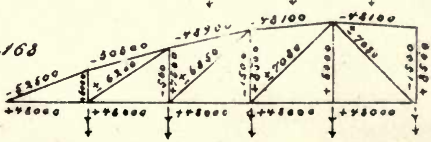
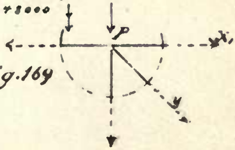


fig.169



$$0 = -V_7 . 2 - 1000 . 2 - 5000 . 2;$$

$$V_7 = -6000 \text{ kil.};$$

$$0 = -x_8 . 0{,}875 - 1000 \left(\tfrac{1}{8} + \dots \tfrac{7}{8}\right) 2 - 5000 \tfrac{1}{8} + \dots \tfrac{7}{8}) 2;$$

$$x_8 = -48000 \text{ kil.};$$

$$0 = z_8 . 0{,}8 - 1000 \left(\tfrac{1}{8} + \dots \tfrac{7}{8}\right) 2 - 5000 \left(\tfrac{1}{8} + \dots \tfrac{7}{8}\right) 2;$$

$$z_8 = +52500 \text{ kil.}$$

156.] In the skeleton (Fig. 156) will be found the combined results.

Comparing the equations with the skeleton, we find the most
157.] strain in a certain diagonal, say y_3, in the third panel, when all the apexes to the right are loaded, and the most compression when all the apexes to the left are loaded. (Fig. 157.)

158.] In the same panel, when the diagonal is reversed or replaced by y_6, the strain will be like y_6. (Fig. 158.)

When both diagonals, y_3 and y_6, in panel 3, are existing and constructed like tie-rods, then each one separately will only be
159.] strained by a load producing tension. Meanwhile the other is inactive, and in this case only the positive strains of y come into consideration. So for y_3 and y_6 in panel 3 (Fig. 159), and in the same manner with the other diagonals, as in
Plate 26, Fig. 160.] the skeleton. (Fig. 160.)

For the verticals only the greatest negative (—) strains come into consideration. Because the diagonals have tensile strain, there can be compression only in the verticals.

In a construction with vertical tie-rods and diagonal braces, for the braces only the greatest negative (—) strains, and for the verticals the positive (+) strains, come into consideration. The only compression in the verticals results from the direct load, varying between 1000 kil. and 1000 + 5000 kil., and is, therefore, the maximum compression in the verticals,

$$V = -6000 \text{ kil.}$$

In this case the inability of diagonals for tensile strain is to be represented by double lines. Also for a girder with single
161, 162, 163.] diagonal, but symmetrical system, the compression of the vertical at the centre will be only — 6000; the other strains all resulting from the first calculation.

Here may follow a combination of different cases in Figs. 161, 162 and 163.

When in the first skeleton (Fig. 156) the symbols + and — are reversed, it represents the strains for a parabolic girder, with the horizontal flange on below; and the variations of this case can be made in the same manner as before. (See Figs. 164 to 168.)

164 to 168.]

Remark.—Two peculiarities we observe by the calculation of parabolic girders. First, the strength in the horizontal chords or flanges is the largest with a full load, and is the same all over. Second, the strain in each diagonal is with a full load $= 0$.

This last fact presents itself from the first theorem, for $x = x_1$ in Fig. 169 is only possible when $y = 0$, otherwise the horizontal component of y would enlarge or diminish x or x_1, and this will be true also in case the width of panels should be different, when only the apexes are in a parabolic curve. This shows the necessity for the application of the rule to calculate the maximum and minimum strain as prescribed in the example.

169.]

[Plates 25 and 26—embracing Figs. 150 to 169.]

D. THE ARCHED TRUSS.

Plate 27, Fig. 170.] The following calculations are for the roof of the Central Depot at Birmingham:

Girder, 208 feet in length. (Fig. 170.)

13 panels, each 16 feet in horizontal length, and 24 feet in depth.

The distances of top intersections to the distances of bottom intersections from the horizontal line are as 40 to 16, or $= 2\frac{1}{2}$ to 1.

The distance of main rafters $= 24$ feet; weight and pressure of snow and wind $= 40$ lbs. per square foot of horizontal projection.

$$208 \times 24 \times 40 = 199680 \text{ lbs. (the weight of load),}$$

or for each panel =

$$\frac{199680}{13} = 15360 \text{ lbs., or } 7{,}5 \text{ tons.}$$

The weight of structure = 1,5 tons for each panel.

The pressure, D, on the abutment, will be

$$D = (1,5 + 7,5)\left(\tfrac{1}{13} + \tfrac{2}{13} + \tfrac{3}{13} + \ldots + \tfrac{11}{13} + \tfrac{12}{13}\right) = 9 \times \tfrac{78}{13} = 54 \text{ tons.}$$

171.] Therefore, per example for x_4, in Fig. 171, when the length of 16 feet is taken as unit, or = 1,

$$0 = x_4 . 1,205 + 54 \times 4 - (7,5 + 1,5)\,(1 + 2 + 3) \text{ (rot. round } M\text{)};$$

$$x_4 = - 134,4 \text{ tons};$$

and for z_4,

$$0 = - z_4 . 1,055 + 54 \times 3 - (7,5 + 1,5)\,(1 + 2) \text{ (rot. } r . N\text{)};$$

$$z_4 = + 128,0 \text{ tons.}$$

In the same manner,

$0 = x_1 . 0,347 + 54 \times 1,$ or $x_1 = - 155,6$ t.

$0 = - z_1 . 0,41 + 54 \times 1,$ or $z_1 = + 131,7$ t.

$0 = x_2 . 0,672 + 54 \times 2 - 9 \times 1,$ or $x_2 = - 147.3$ t.

$0 = - z_2 . 0,415 + 54 \times 1,$ or $z_2 = + 130,2$ t.

$0 = x_3 . 0,963 + 54 \times 3 - 9 . (1 + 2)$ or $x_3 = - 140,2$ t.

$0 = - z_3 . 0,767 + 54 \times 2 - 9 \times 1,$ or $z_3 = + 129,1$ t.

$0 = x_5 . 1,382 + 54 \times 5 - 9\,(1 + 2 + 3 + 4),$ or $x_5 = - 130,2$ t.

$0 = - z_5 . 1,272 + 54 \times 4 - 9\,(1 + 2 + 3)$ or $z_5 = + 127,3$ t.

$0 = x_6 . 1,481 + 54 \times 6 - 9\,(1 + 2 + 3 + 4 + 5),$ or $x_6 = - 127,6$ t.

$0 = - z_6 . 1,419 + 54 \times 5 - 9\,(1 + 2 + 3 + 4),$ or $z_8 = + 126,9$ t.

$0 = x_7 . 1,491 + 54 \times 7 - 9\,(1 + 2 + 3 + 4 + 5 + 6),$ or $x_7 = - 126,7$ t.

$0 = - z_7 . 1,491 + 54 \times 6 - 9\,(1 + 2 + 3 + 4 + 5),$ or $z_7 = + 126,7$ t.

$0 = x_8 . 1,41 + 54 \times 8 - 9\,(1 + 2 + \ldots 7),$ or $x_8 = - 127,6$ t.

$0 = - z_8 . 1,489 + 54 \times 7 - 9\,(1 + 2 + \ldots 6),$ or $z_8 = + 126,9$ t.

$0 = x_9 . 1,244 + 54 \times 9 - 9\,(1 + 2 + \ldots 8),$ or $x_9 = - 130,2$ t.

$0 = - z_9 . 1,414 + 54 \times 8 - 9\,(1 + 2 + \ldots 7),$ or $z_9 = + 127,3$ t.

$0 = x_{10} . 1{,}004 + 54 \times 10 - 9 (1 + 2 + \ldots 9)$, or $x_{10} = - 134{,}4$ t.

$0 = - z_{10} . 1{,}265 + 54 \times 9 - 9 (1 + 2 + \ldots 8)$, or $z_{10} = + 128{,}0$ t.

$0 = x_{11} . 0{,}706 + 54 \times 11 - 9 (1 + 2 + \ldots 10)$, or $x_{11} = - 140{,}2$ t.

$0 = - z_{11} . 1{,}046 + 54 \times 10 - 9 (1 + 2 + \ldots 9)$, or $z_{11} = + 129{,}1$ t.

$0 = x_{12} . 0{,}367 + 54 \times 12 - 9 (1 + 2 + \ldots 11)$, or $x_{12} = - 147{,}3$ t.

$0 = - z_{12} . 0{,}76 + 54 \times 11 - 9 (1 + 2 + \ldots 10)$, or $z_{12} = + 130{,}2$ t.

$0 = x_{13} . 0{,}347 + 54 \times 12 - 9 (1 + 2 + \ldots 11)$, or $x_{13} = - 155{,}6$ t.

$0 = - z_{13} . 0{,}41 + 54 \times 12 - 9 (1 + 2 + \ldots 11)$, or $z_{13} = + 131{,}7$ t.

Remark.—The results noted in Fig. 176 show that the greatest strains in the symmetrical sections of flanges are the same, though the diagonals of one-half of the girder are reversed to the others, and it follows that for the definition of strain in flanges it will be the same if we take the point for rotation in the right or left apex.

This is only possible when the strain in the diagonal $= 0$, and therefore shows us that by a full load, as in parabolic girders, no strain in diagonals exists. Nevertheless, a partial load (from snow or wind or removing of sheeting) being unavoidable, the diagonal connections are a necessity.

Calculation of Strain y in the Diagonals.

172.] For y_4 (see Fig. 172), when the point of rotation, O, in the intersection of x_4 and z_4 and the length OA, found by construction $= 32$ feet, or, for easier calculation, 16 feet $=$ unit (1); therefore $OA = 2$, and the lever for $y_4 = PO = 4{,}68$.

$$0 = y_4 . 4{,}68 - D . 2 + 1{,}5 [(3 + 2) + (2 + 2) + (1 + 2)] + 7{,}5 [(3 + 2) + (2 + 2) + (1 + 2)],$$

and $D = 1{,}5 (\frac{1}{13} + \frac{2}{13} + \ldots \frac{12}{13}) + 7{,}5 (\frac{1}{13} + \frac{2}{13} + \ldots \frac{12}{13})$,

substituted with its members of permanent and variable load on their respective places,

$$0 = y_4 . 4{,}68 - 1{,}5 [(\tfrac{1}{13} + \tfrac{2}{13} + \ldots \tfrac{9}{13}) 2 - (3 + 2 + 1) (1 + \tfrac{2}{13})] - 7{,}5 \tfrac{1}{13} + \tfrac{2}{13} + \ldots \tfrac{9}{13}) + 7{,}5 (3 + 2 + 1) (1 + \tfrac{2}{13}).$$

The solution of this equation shows the member for a permanent load $= 0$, and therefore our equation in its more simple form—

$$0 = -y_4 . 4{,}68 - 7{,}5 \left(\tfrac{1}{13} + \tfrac{2}{13} + \dots \tfrac{9}{13}\right) 2 + 7{,}5 \left(3 + 2 + 1 \left(1 + \tfrac{2}{13}\right)\right),$$

or, according to the rule formerly given,

$$\text{I.}\quad 0 = -y_4 . 4{,}68 - 7{,}5 \left(\tfrac{1}{13} + \tfrac{2}{13} + \dots \tfrac{9}{13}\right) 2;$$

$$\text{II.}\quad 0 = -y_4 . 4{,}68 + 7{,}5 (3 + 2 + 1) \left(1 + \tfrac{2}{13}\right),$$

or $\qquad y_4 = +\,11{,}1$ tons, $\qquad$ and $y_4 = -\,11{,}1$ tons.

In the same way for the other diagonals and the length of lever from construction,

$$0 = y_2 . 0{,}92 - 7{,}5 \left(\tfrac{1}{13} + \tfrac{2}{13} + \dots \tfrac{11}{13}\right) . 0{,}2 + 7{,}5 \left(1 + \tfrac{0{,}2}{13}\right);$$

$$y_2 = +\,8{,}3, \qquad \text{and } y_3 = -\,8{,}3 \text{ tons};$$

$$0 = y_3 . 2{,}52 - 7{,}5 \left(\tfrac{1}{13} + \dots \tfrac{10}{13}\right) 0{,}75 + 7{,}5 (2 + 1) \left(1 + \tfrac{0{,}75}{13}\right);$$

$$y_3 = +\,9{,}5, \qquad \text{and } y_3 = -\,9{,}5 \text{ tons};$$

$$0 = y_5 . 8{,}3 - 7{,}5 \left(\tfrac{1}{13} + \dots \tfrac{8}{13}\right) 5 + 7{,}5 (4 + 3 + 2 + 1) \left(1 + \tfrac{5}{13}\right);$$

$$y_5 = +\,12{,}6, \qquad \text{and } y_5 = -\,12{,}6 \text{ tons};$$

$$0 = y_6 . 17{,}6 - 7{,}5 \left(\tfrac{1}{13} + \dots \tfrac{7}{13}\right) 15 + 7{,}5 (5 + 4 + 3 + 2 + 1) \left(1 + \tfrac{15}{13}\right);$$

$$y_8 = +\,13{,}8, \qquad \text{and } y_8 = -\,13{,}8 \text{ tons}.$$

The point of rotation, O, is for the diagonal of the middle panel in infinite distance. (See girders with horizontal top and bottom flanges.)

The sinus of the angle formed by y_7 and a horizontal line = 0,831, leading to the equation in its most simple form (as before explained in examples for girders with horizontal top and bottom flanges).

$$0 = y_7 . 0{,}831 - 7{,}5 \left(\tfrac{1}{13} + \dots \tfrac{6}{13}\right) + 7{,}5 (6 + \dots 1) \tfrac{1}{13};$$

$$y_7 = +\,14{,}6, \qquad \text{and } y_7 = -\,14{,}6 \text{ tons}.$$

For the equations now following, the point of rotation will be on the opposite side; therefore the symbols of moments are reversed.

$$0 = -y_8 . 16{,}1 + 7{,}5 \left(\tfrac{1}{13} + \dots \tfrac{5}{13}\right) 28 - 7{,}5 (7 + \dots + 1) \left(\tfrac{28}{13} - 1\right);$$

$$y_8 = +\,15{,}0, \qquad \text{and } y_8 = -\,15{,}0 \text{ tons};$$

$$0 = -y_9 . 7{,}1 + 7{,}5 \left(\tfrac{1}{13} + \dots \tfrac{4}{13}\right) 18 - 7{,}5 (8 + \dots + 1) \left(\tfrac{18}{13} - 1\right);$$

$$y_9 = +\,14{,}6, \qquad \text{and } y_9 = -\,14{,}6 \text{ tons};$$

$0 = -y_{10} . 3{,}68 + 7{,}5 (\frac{1}{13} + \frac{2}{13} + \frac{3}{13}) 15 - 7{,}5 (9 + \ldots 1) (\frac{15}{13} - 1);$

$$y_{10} = +14{,}1, \quad \text{and } y_{10} = -14{,}1 \text{ tons};$$

$0 = -y_{11} . 1{,}82 + 7{,}5 (\frac{1}{13} + \frac{2}{13}) . 13{,}75 - 7{,}5 (10 + \ldots 1) (\frac{13{,}75}{13} - 1);$

$$y_{11} = +13{,}0, \quad \text{and } y_{11} = -13{,}0 \text{ tons};$$

$0 = -y_{12} . 0{,}65 + 7{,}5 \times \frac{1}{13} \times 13{,}2 - 7{,}5 (11 + \ldots 1) (\frac{13{,}2}{13} - 1);$

$$y_{12} = +11{,}6, \quad \text{and } y_{12} = -11{,}6 \text{ tons}.$$

Calculation of Strain in the Verticals V.

173.] For V_1, when the point of rotation in the intersection of x_1 and z_1, which by construction $= 0{,}1$ to the right of A (Fig. 173),

$$0 = -V_1 . 0{,}9 + D . 0{,}1,$$

or $0 = -V_1 . 0{,}9 + 1{,}5 (\frac{1}{13} + \frac{2}{13} + \ldots \frac{12}{13}) 0{,}1 + 7{,}5 (\frac{1}{13} + \frac{2}{13} + \ldots \frac{12}{13}) 0{,}1;$

$$V_1 = +6 \text{ tons}.$$

174.] For V_2, the point of rotation in the intersection of x_2 and $z_2 = 0{,}06$ to the right of A. (Fig. 174.)

$$0 = -V_2 . 1{,}94 + D . 0{,}06 + 1{,}5 \times 0{,}94 + 7{,}5 \times 0{,}94;$$

$$0 = -V_2 . 1{,}94 + 1{,}5 [(\tfrac{1}{13} + \ldots \tfrac{11}{13}) 0{,}06 + (1 - \tfrac{0{,}06}{13})]$$
$$+ 7{,}5 (\tfrac{1}{13} + \ldots \tfrac{11}{13}) 0{,}06 + 7{,}5 (1 - \tfrac{0{,}06}{13});$$

$$V_2 = +6 \text{ tons}.$$

175.] For the equations now following, the point of rotation, O, will appear to the left of $A = 0{,}214$ for the intersection of x_3 and z_4, and for V_3 (Fig. 175) we have

$$0 = -V_3 . 3{,}214 - 1{,}5 [(\tfrac{1}{13} + \ldots \tfrac{10}{13}) 0{,}214 - (2 + 1) (1 + \tfrac{0{,}214}{13})]$$
$$- 7{,}5 (\tfrac{1}{13} + \ldots \tfrac{10}{13}) 0{,}214 + 7{,}5 (2 + 1) (1 + \tfrac{0{,}214}{13});$$

and omitting at one time the positive and at another time the negative members of the movable load, we find

$$V_3 \text{ (max.)} = +8{,}1 \text{ tons}$$
$$V_3 \text{ (min.)} = -1{,}1 \text{ ``}$$

and $V_3 = +6$ tons when no member is omitted, which will be the

same for the other verticals, or $V_4 = V_5 = V_6 \ldots = 6$ tons; *i. e.*, for a full load of the truss.

In the same way

$$0 = -V_4 \,.\, 4{,}91 - 1{,}5 \left[\left(\tfrac{1}{13} + \ldots \tfrac{9}{13}\right) 0{,}91 - (3+2+1)\left(1 + \tfrac{0{,}91}{13}\right)\right]$$
$$- 7{,}5 \left(\tfrac{1}{13} + \ldots \tfrac{9}{13}\right) 0{,}91 + 7{,}5 \,(3+2+1)\left(1 + \tfrac{0{,}91}{13}\right);$$
$$V_4\,(\text{max.}) = +\,10{,}8;$$
$$V_4\,(\text{min.}) = -\,3{,}8;$$

$$0 = -V_5 \,.\, 7{,}5 - 1{,}5 \left[\left(\tfrac{1}{13} + \ldots \tfrac{8}{13}\right) 2{,}5 - (4 + \ldots 1)\left(1 + \tfrac{2{,}5}{13}\right)\right]$$
$$- 7{,}5 \left(\tfrac{1}{13} + \ldots \tfrac{8}{13}\right) 2{,}5 + 7{,}5 \,(4 + \ldots 1)\left(1 + \tfrac{2{,}5}{13}\right);$$
$$V_5 = +\,12{,}9;$$
$$V_5 = -\,5{,}9;$$

$$0 = -V_6 \,.\, 12{,}6 - 1{,}5 \left[\left(\tfrac{1}{13} + \ldots \tfrac{7}{13}\right) 6{,}6 - (5 + \ldots 1)\left(1 + \tfrac{6{,}6}{13}\right)\right]$$
$$- 7{,}5 \left(\tfrac{1}{13} + \ldots \tfrac{7}{13}\right) 6{,}6 + 7{,}5 \,(5 + \ldots 1)\left(1 + \tfrac{6{,}6}{13}\right);$$
$$V_6 = +\,14{,}5;$$
$$V_6 = -\,7{,}5;$$

$$0 = -V_7 \,.\, 31{,}5 - 1{,}5 \left[\left(\tfrac{1}{13} + \ldots \tfrac{6}{13}\right) 24{,}5 - (6 + \ldots 1)\left(1 + \tfrac{24{,}5}{13}\right)\right]$$
$$- 7{,}5 \left(\tfrac{1}{13} + \ldots \tfrac{6}{13}\right) 24{,}5 + 7{,}5 \,(6 + \ldots 1)\left(1 + \tfrac{24{,}5}{13}\right);$$
$$V_7 = +\,15{,}4;$$
$$V_7 = -\,8{,}4.$$

In the equations now following, the point of rotation on the opposite side gives,

$$0 = V_8 \,.\, 60 + 1{,}5 \left[\left(\tfrac{1}{13} + \ldots \tfrac{5}{13}\right) 68 - (7 + \ldots 1)\left(\tfrac{68}{13} - 1\right)\right]$$
$$+ 7{,}5 \left(\tfrac{1}{13} + \ldots \tfrac{5}{13}\right) 68 - 7{,}5 \,(7 + \ldots 1)\left(\tfrac{68}{13} - 1\right);$$
$$V_8 = +\,15{,}8;$$
$$V_8 = -\,8{,}8;$$

$$0 = V_9 \,.\, 13{,}5 + 1{,}5 \left[\left(\tfrac{1}{13} + \ldots \tfrac{4}{13}\right) 22{,}5 - (8 + \ldots 1)\left(\tfrac{22{,}5}{13} - 1\right)\right]$$
$$+ 7{,}5 \left(\tfrac{1}{13} + \ldots \tfrac{4}{13}\right) 2{,}25 - 7{,}5 \,(8 + \ldots 1)\left(\tfrac{2{,}25}{13} - 1\right);$$
$$V_9 = +\,15{,}6;$$
$$V_9 = -\,8{,}6;$$

$$0 = V_{10} . 6{,}43 + 1{,}5 \left[\left(\tfrac{1}{13} + \tfrac{2}{13} + \tfrac{3}{13}\right) 16{,}43 - (9 + 1)\left(\tfrac{16{,}43}{13} - 1\right)\right]$$
$$+ 7{,}5 \left(\tfrac{1}{13} + \tfrac{2}{13} + \tfrac{3}{13}\right) 16{,}43 - 7{,}5\,(9 + \ldots 1)\left(\tfrac{16{,}43}{13} - 1\right);$$
$$V_{10} = + 14{,}8;$$
$$V_{10} = - 7{,}8;$$
$$0 = V_{11} . 3{,}3 + 1{,}5 \left[\left(\tfrac{1}{13} + \tfrac{2}{13}\right) 14{,}3 - (10 + \ldots 1)\, \tfrac{14{,}3}{13} - 1)\right]$$
$$+ 7{,}5 \left(\tfrac{1}{13} + \tfrac{2}{13}\right) 14{,}3 - 7{,}5\,(10 + \ldots 1)\left(\tfrac{14{,}3}{13} - 1\right);$$
$$V_{11} = + 13{,}5;$$
$$V_{11} = - 6{,}5;$$
$$0 = V_{12} . 1{,}385 + 1{,}5 \left[\tfrac{1}{13} \times 13{,}385 - (11 + \ldots 1)\left(\tfrac{13{,}385}{13} - 1\right)\right]$$
$$+ 7{,}5 \times \tfrac{1}{13} \times 13{,}385 - 7{,}5\,(11 + \ldots 1)\left(\tfrac{13{,}385}{13} - 1\right);$$
$$V_{12} = + 11{,}6;$$
$$V_{12} = - 4{,}6 \text{ tons.}$$

The strains, V, in the verticals have been calculated under the supposition that the whole permanent load (weight of structure) is charged to the upper apexes. In reality, such is not true; and in consideration that about one-third of this load should be transmitted to the lower apexes, we increase the strains in verticals (in this case tie-rods) for 0,5 tons each, which changes the above results to the following:

$$V_1 \text{ (max.)} = + 6{,}5 \text{ tons};$$
$$V_2 \text{ (max.)} = + 6{,}5 \quad \text{“}$$
$$V_3 \begin{cases} \text{(max.)} = + \ \ 8{,}6 \\ \text{(min.)} = - \ \ 0{,}6 \end{cases} V_3 = + 6{,}5;$$
$$V_4 \begin{cases} \text{(max.)} = + 11{,}3 \\ \text{(min.)} = - \ \ 3{,}3 \end{cases} V_4 = + 6{,}5;$$
$$V_5 \begin{cases} \text{(max.)} = + 13{,}4 \\ \text{(min.)} = - \ \ 5{,}4 \end{cases} V_5 = + 6{,}5;$$
$$V_6 \begin{cases} \text{(max.)} = + 15{,}0 \\ \text{(min.)} = - \ \ 7{,}0 \end{cases} V_6 = + 6{,}5;$$
$$V_7 \begin{cases} \text{(max.)} = + 15{,}9 \\ \text{(min.)} = - \ \ 7{,}9 \end{cases} V_7 = + 6{,}5;$$
$$V_8 \begin{cases} \text{(max.)} = + 16{,}3 \\ \text{(min.)} = - \ \ 8{,}3 \end{cases} V_8 = + 6{,}5;$$

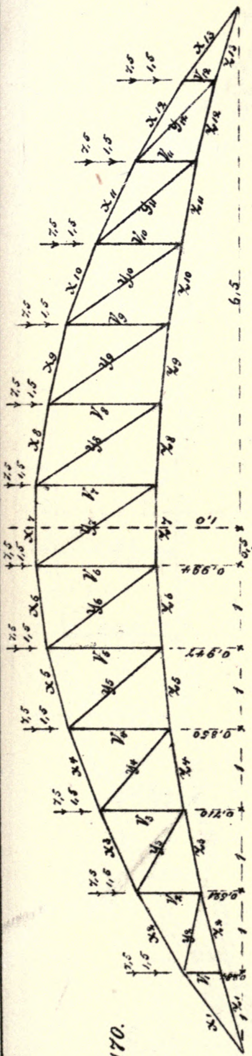


fig. 170.

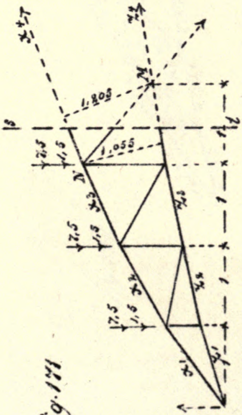


fig. 171

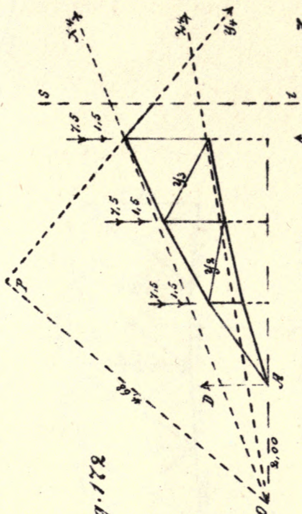


fig. 172

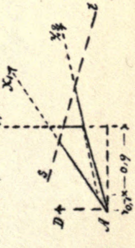


fig. 173

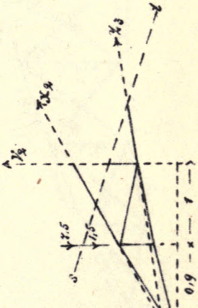


fig. 174

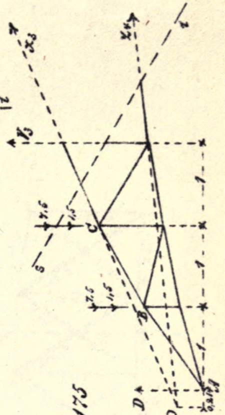


fig. 175

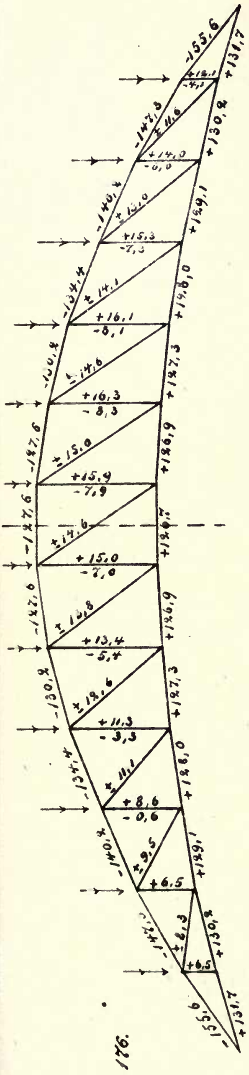


fig. 176.

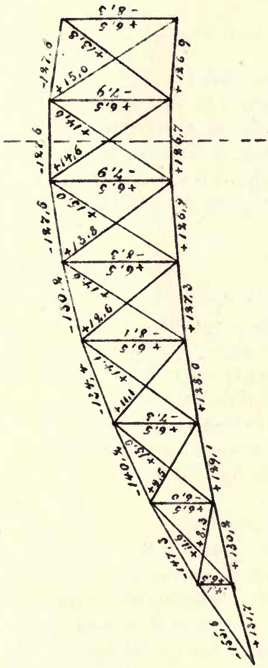


fig. 177.

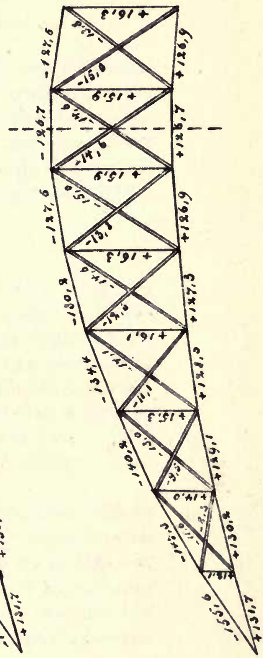


fig. 178.

$$V_9 \begin{cases} (\text{max.}) = +\ 16{,}1 \\ (\text{min.}) = -\ \ 8{,}1 \end{cases} V_9 = +\ 6{,}5;$$

$$V_{10} \begin{cases} (\text{max.}) = +\ 15{,}3 \\ (\text{min.}) = -\ \ 7{,}3 \end{cases} V_{10} = +\ 6{,}5;$$

$$V_{11} \begin{cases} (\text{max.}) = +\ 14{,}0 \\ (\text{min.}) = -\ \ 6{,}0 \end{cases} V_{11} = +\ 6{,}5;$$

$$V_{12} \begin{cases} (\text{max.}) = +\ 12{,}1 \\ (\text{min.}) = -\ \ 4{,}1 \end{cases} V_{12} = +\ 6{,}5.$$

The results are combined in Fig. 176, and show that the strains in arches are independent of the direction of diagonals, because to both sides of the centre line their strain is the same, though the direction of the diagonal is not symmetrical to the centre line; therefore, for the definition of strain in x or z, it will make no difference if we choose in certain panels the left or right apex for the point of rotation.

Transformations.

From the results above we see that for a single diagonal system the diagonals are strained both for tension and compression. For a girder with a single diagonal, but symmetrical system, we pursue the same course as in the parabolic girder, and it needs no further explanation to form from the preceding figure a girder with symmetrical diagonals, sloping right downward for one and right upward for the other half of girder; or, reversed, right upward for one and right downward for the other half of girder.

[Plate 28, Fig. 176.]

The transformation for a girder with crossed diagonals will be apparent from Skeleton 177, and it is only necessary to remark that, where the diagonals as tie-rods are constructed, for the verticals only the greatest compressive (—) strains come into consideration—observing from this that, per example, the minimum strain of V_4 will be replaced by the greater minimum strain of V_9 as its symmetrical opposite.

177.]

When the diagonals (for instance, by timbers) are constructed like braces as in Skeleton 178, designed by double lines, for the verticals only the maximum or greatest (+) strains come into consideration. The verticals are tie-rods in this case, acting for tensile strain.

178.]

[Plates 27 and 28—embracing Figs. 170 to 178.]

E. THRUST CONSTRUCTION.

To form the equations of equilibrium, in the preceding calculations the reactive force of supports has formed an essential part, in place of which, by the thrust construction, the resulting force at the vertex comes into consideration. To define its intensity, a single weight, W, may be supposed to rest at E (Fig. 179) on an arch-shaped girder, fixed at the heels, A and B, and butted at F.

[Plate 29, Fig. 179.]

The weight, W, at a distance, x, from the right support, results in the force AW, intersecting the vertex F, and in the force WB toward the right support.

When R is the intensity in the direction AW, its components are H and V (downward) for the left section, and H and V (upward) for the right section (Fig. 180[a]); and for the equilibrium we have

[180[a].]

left section, $\quad 0 = V.10 - H.4$ (rot. A);

right section, $\quad 0 = V.10 + H.4 - Wx$ (rot. B),

in which, by addition,

$$0 = V.20 - Wx, \qquad \text{or } V = \frac{Wx}{20},$$

and by subtraction,

$$0 = H.8 - Wx, \qquad \text{or } H = \frac{Wx}{8}.$$

When $x = 10$ feet (*i. e.*, the weight, W, removed to the centre of the girder, Fig. 180[a]), then

$$V = \frac{W.10}{20}, \qquad \text{or } V = \frac{W}{2};$$

$$H = \frac{W.10}{8}, \qquad \text{or } H = \tfrac{5}{4}W,$$

(being for a weight, $W = 100000$ lbs.);

$$V = 50000 \text{ lbs.}, \qquad \text{and } H = 125000 \text{ lbs.}$$

Thus, having defined the forces at the vertex for a single weight, W, at the centre, for the diagonals we have the following equations of equilibrium, a cut, s_4t_4, being supposed to separate the members next to the left support.

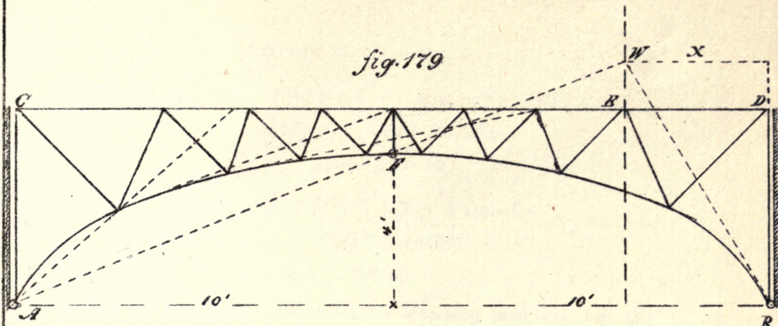


fig. 179

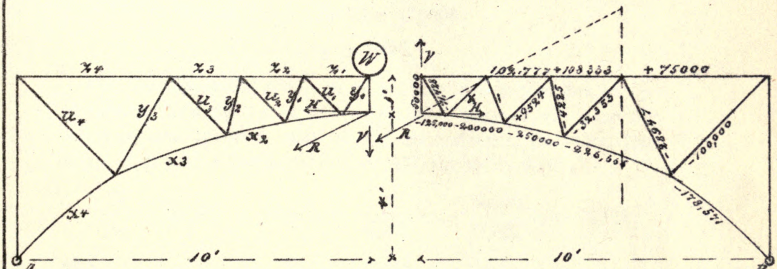


fig. 180a

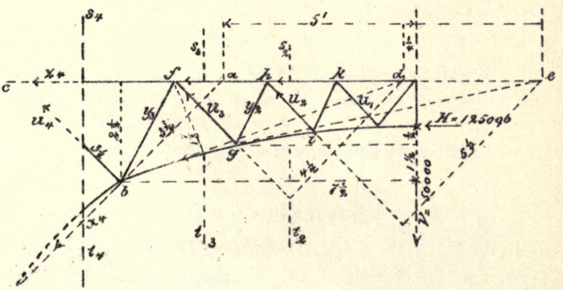


fig. 180b

180b.] $0 = u_4 . 3\frac{3}{4} + H \times 1 + V \times 5$ (rot. a, Fig. 180b);

$$0 = u_4 . 3\tfrac{3}{4} + 125000 + 250000;$$

$$u_4 = -100000.$$

$$0 = u_3 . 4\tfrac{1}{4} + H \times 1 + V \times \tfrac{1}{4} \text{ (rot. } d);$$

$$0 = u_3 . 4\tfrac{1}{4} + 125000 + 12500;$$

$$u_3 = -32353.$$

For u_2, the intersection, e, of the adjoining members, z of the upper and x of the lower string, is to the right of the centre; therefore, for the equilibrium,

$$0 = u_2 . 5\tfrac{1}{4} + H \times 1 - V \times 3\tfrac{1}{2} \text{ (rot. } e);$$

$$0 = u_2 . 5\tfrac{1}{4} + 125000 - 175000;$$

$$u_2 = +9524.$$

For the diagonals u_1 and y_0 we suppose the intersection to be in infinite distance (*i. e.*, z and x parallel); then H will have no leverage, and therefore will be without influence, and when the angle of y_0 with a horizontal line $= 45°$, $\sin 45° = 0{,}707$,

$$-\frac{V}{y_0} = \sin 45, \qquad \text{or } y_0 = -\frac{V}{0{,}707} = -\frac{50000}{0{,}707} = -71428;$$

also, $u_1 = +71428.$

For y_2 we have

$$0 = -y_2 \times 5\tfrac{7}{8} - V \times 3 + H \times 1 \text{ (rot. } e);$$

$$y_2 = -4255;$$

also, $y_3 = +28947.$

The other diagonals, y, will be found in the same manner, and will be in numerical value the same as the diagonals u, in case their angle with the horizontal line is the same.

For the strains z in the upper string the rotation will be at the lower apexes; so for z_4,

$$0 = -z_4 \times 2\tfrac{1}{2} - H \times 1\tfrac{1}{2} + V \times 7\tfrac{1}{2} \text{ (rot. } b);$$

$$0 = -z_4 \times 2\tfrac{1}{2} - 187500 + 375000;$$

$$z_4 = +75000;$$

$$0 = -z_3 \times 1\tfrac{1}{2} - H \times \tfrac{1}{2} + V \times 4\tfrac{1}{2} \text{ (rot. } g);$$

$$z_3 = +108333;$$

$$0 = -z_2 \times 1\tfrac{1}{8} - H \times \tfrac{1}{8} + V \times 2\tfrac{5}{8} \text{ (rot. } i);$$

$$z_2 = 102777.$$

In the same way, but with the rotation in the upper apexes, we have for the forces x in the lower stringer,

$$0 = x_4 \times 3\tfrac{1}{2} + H \times 1 + V \times 10 \text{ (rot. } c);$$

$$x_4 = -178571;$$

$$0 = x_3 \times 1\tfrac{7}{8} + H \times 1 + V \times 6 \text{ (rot. } f);$$

$$x_3 = -226666;$$

$$0 = x_2 \times 1\tfrac{1}{4} + H + V \times 3\tfrac{3}{4} \text{ (rot. } h);$$

$$x_2 = -250000;$$

$$0 = x_1 \times 1\tfrac{1}{8} + H \times 1 + V \times 2 \text{ (rot. } k);$$

$$x_1 = -200000.$$

The results are given in the right section of Fig. 180[a].

Plate 30, Fig. 181.] For a combined (permanent and rolling) load another example will explain the definition of strains:

Span = 72 feet, or 24 metres; (Fig. 181.)

permanent load = 2 tons for each apex;

rolling load = 6 tons for each apex.

To ascertain the maximum strain of the single members we again first take into consideration what influence a single load, Q, upon the structure will have.

For a single weight, Q, to the right of the centre line, and the produced pressure, R, toward the left abutment, it will be observed that the reaction of the left support is in the line AS, the prolongation of which intersects in P with a vertical line in Q, the reaction of A_1 being also directed toward P.

The pressure at the link in the vertex S we divide in its horizon-
182.] tal and vertical components, H and V, and when the moments are formed each for one-half of the structure in relation to their respective supports, we find from Fig. 182,

$$0 = V.12 + H.4 - Q.3 \text{ for the right section (rot. } A_1);$$

$$0 = V.12 - H.4 \text{ for the left section (rot. } A).$$

These equations, when at one time added and at another subtracted, result in

$$V = \frac{Q}{8}, \quad \text{and } H = \tfrac{3}{8} Q.$$

As by this mode of observation we are enabled to ascertain the influence of a weight, Q, upon the whole system, for the definition of strain in a certain member of the structure we make a cut, *st*, and form for the considered section the equation of moments.

Definition of Strain x in the Horizontal Flanges.

For a point of rotation we take the foot, E, of the diagonal. (B, E, Fig. 182.)

Each weight to the left of the centre line produces a pressure in the vertex, S, the direction of which is from A_1 toward S, to keep the section in its position.

The components of this pressure (H and V) aim to turn to the left round A, similar to the strain x_1 itself, thereby making x_1 compressive.

Each weight to the right of the centre line produces for the left part a pressure in the vertex, the direction of which is through the point of rotation, E, and for this reason has no influence.

For the greatest compression, therefore, we consider one-half of the girder, containing the flange in question, charged with a full load. The other half can be either loaded or unloaded without influence, as already stated.

183.] Both halves being loaded, we have, from Fig. 183, for the pressure in the vertex,

$$0 = V.12 + H.4 - 4 \times 12 - 8\,(9 + 6 + 3)\ (b) \text{ rot. } A_1;$$

$$0 = V.12 - H.4 + 4 \times 12 + 8\,(9 + 6 + 3)\ (a) \text{ rot. } A;$$

$$V = 0, \quad \text{and } H = 48.$$

184.] From this for x_1 (rot. $r.E$, Fig. 184),

$$0 = -x_1 . 3{,}5 - 48 \times 3 + 8\,(3 + 6) + 4 \times 9;$$

$$x_1 = -10{,}29 \text{ tons};$$

and in the same way,

$$0 = -x_2 . 2{,}5 - 48 \times 2 + 8 \times 3 + 4 \times 6\,;$$

$$x_2 = -19{,}2 \text{ tons}\,;$$

$$0 = -x_3 . 1{,}5 - 48 \times 1 + 4 \times 3\,;$$

$$x_3 = -24 \text{ tons}\,;$$

$$0 = -x_4 . 0{,}5\,;$$

$$x_4 = 0.$$

Definition of Strain y in the Diagonals.

When for a section (Fig. 186) the strain in the diagonal, y_2, is to be calculated, the first thing is to define the strain in the vertex.

185.] For y_2, maximum, the apexes 3 and 4 ought to be loaded. The others are indifferent, and we have for the strain in the vertex (Fig. 185),

$$0 = -V . 12 + H . 4 - 1 \times 12 - 2\,(9 + 6 + 3)\ (\text{rot. } A_1)\,;$$

$$0 = -V . 12 - H . 4 + 1 \times 12 + 2\,(9 + 6 + 3) + 6\,(9 + 6)\ (\text{rot. } A)\,;$$

$$V = 3{,}75, \qquad \text{and } H = 23{,}25,$$

which gives for Fig. 186 the equation of equilibrium.

$$0 = y_2 . 6{,}72 + 23{,}25 \times 0{,}5 + 3{,}75 \times 1{,}5 - 1 \times 1{,}5 - 8\,(4{,}5 + 7{,}5\ (\text{rot. } F)\,;$$

$$y_2\ (\text{max.}) = +\,11{,}94 \text{ tons.}$$

186.] For y_2, (min.), the apexes 3 and 4 ought to be unloaded, the second loaded. The others again are indifferent, and may be unloaded.

187.] For the strain in the vertex is from the equations formed from Fig. 187,

$$0 = -V . 12 + H . 4 - 1 \times 12 - 2\,(9 + 6 + 3)\ (\text{rot. } r . A_1)\,;$$

$$0 = -V . 12 - H . 4 + 1 \times 12 + 2\,(9 + 6 + 3) + 6 \times 3\ (\text{rot. } r . A)\,;$$

$$V_0 = 0{,}75, \qquad \text{and } H = 14{,}25\,;$$

Plate 31, Fig. 188.] And thus we find for the moments upon the section (Fig. 188),

fig. 181

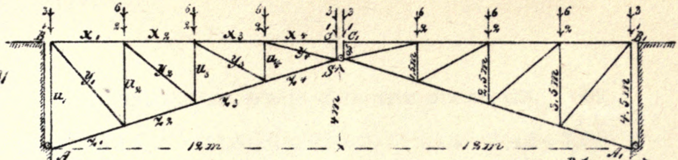


fig. 182

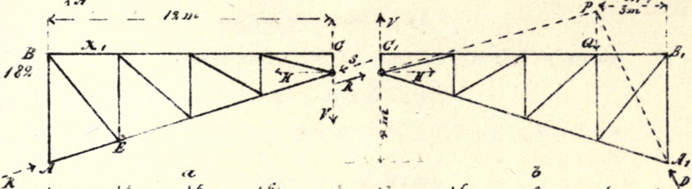


fig. 183

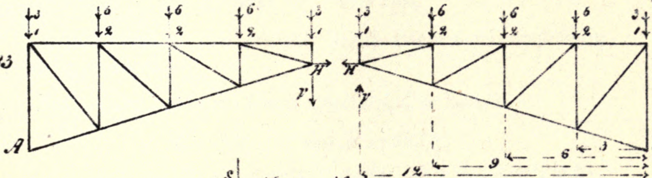


fig. 184

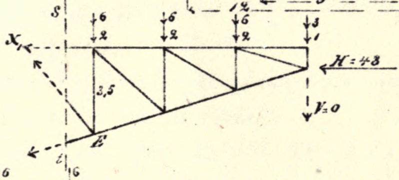


fig. 185

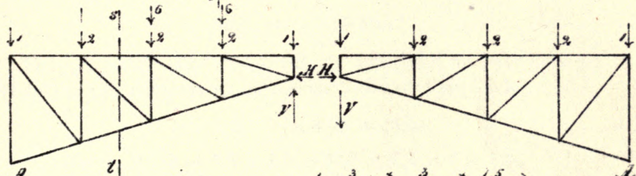


fig. 186

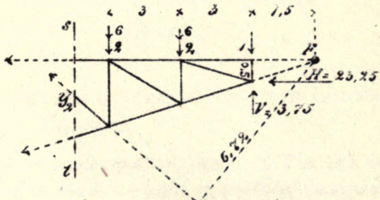
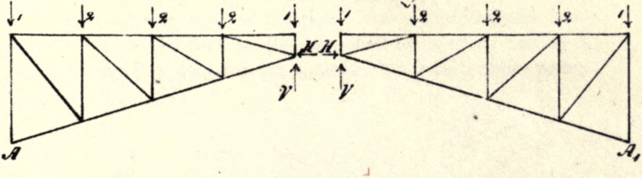


fig. 187



$$0 = y_2 . 6{,}72 + 0{,}75 \times 1{,}5 + 14{,}25 \times 0{,}5 - 1 \times 1{,}5 - 2\,(4{,}5 + 7{,}5);$$

$$y_2 \text{ (min.)} = + 2{,}75 \text{ tons.}$$

In the same manner we find for y_1 (max.) the following equations:

With a full load, $V = 0$, and $H = 48$;

therefore, $0 = y_1 . 10{,}25 + 48 \times 0{,}5 - 4 \times 1{,}5 - 8\,(4{,}5 + 7{,}5 + 10{,}5)$;

$$y_1 \text{ (max.)} = + 15{,}8 \text{ tons.}$$

y_1 (min.) does not exist, because in this diagonal no compression can be produced.

For y_3 (max.) only the fourth apex should be loaded.

$$V = 2{,}25, \quad \text{and } H = 18{,}75;$$

$$0 = y_3 . 3{,}35 + 2{,}25 \times 1{,}5 + 18{,}75 \times 0{,}5 - 1 \times 1{,}5 - 8 \times 4{,}5;$$

$$y_3 \text{ (max.)} = + 7{,}39 \text{ tons.}$$

For y_3 (min.), when only the second and third apexes are loaded,

$$V = 2{,}25, \quad \text{and } H = 18{,}75;$$

$$0 = y_3 . 3{,}35 + 2{,}25 \times 1{,}5 + 18{,}75 \times 0{,}5 - 1 \times 1{,}5 - 2 \times 4{,}5;$$

$$y_3 \text{ (min.)} = - 0{,}67 \text{ tons.}$$

For y_4, a compressive strain will not exist, and we have only y_4 (min.) to calculate, for which Apexes 2, 3 and 4 are loaded

$$V = 4{,}5, \quad \text{and } H = 25{,}5;$$

$$0 = y_4 . 0{,}738 + 4{,}5 \times 1{,}5 + 25{,}5 \times 0{,}5 - 1 \times 1{,}5;$$

$$y_4 \text{ (min.)} = - 24{,}4 \text{ tons.}$$

CALCULATION OF THE TENSILE STRAINS z IN THE LOWER FLANGES.

For z_3 we find, after short contemplation, that st, in Fig. 189, is
189.] the separating line, in which a weight, q, produces no strain in z_3. Every load to the right of this line produces compression, and every load to the left produces tension, making z_3 positive; therefore we have for z_3 (max.) the strain in the vertex from Fig. 189,

$0 = -V.12 + H.4 - 1 \times 2 - 2\,(9 + 6 + 3)$ (rot. $r.A_1$);

$0 = -V.12 - H.4 + 1 \times 12 + 2\,(9 + 6 + 3) + 6\,(6 + 3$ (rot. $r.A$);

$$V = 2{,}25, \qquad \text{and } H = 18{,}75;$$

190.] and from Fig. 190, for the equations of equilibrium,

$0 = z_3.2{,}37 - 2{,}25 \times 6 + 18{,}75 \times 0{,}5 + 1 \times 6 + 2 \times 3$ (rot. I);

$$z_3\ (\text{max.}) = -3{,}32 \text{ tons.}$$

191.] For z_3 (min.), from Fig. 191,

$0 = V.12 + H.4 - 1 \times 12 - 2\,(9 + 6 + 3) - 3 \times 12 - 6\,(9 + 6 + 3)$ (rot. $r.A_1$);

$0 = V.12 - H.4 + 1 \times 12 + 2\,(9 + 6 + 3) + 3 \times 12 + 6 \times 9$ (rot. $r.A$);

$$V = 2{,}25, \qquad \text{and } H = 41{,}25;$$

192.] and from this,

$0 = z_3.2{,}37 + 2{,}25 \times 6 + 41{,}25 \times 0{,}5 + 4 \times 6 + 8 \times 3$ (rot. $r.I$, Fig. 192);

$$z_3\ (\text{min.}) = -34{,}6 \text{ tons.}$$

In the same manner for z_1, for which the separating line in the first apex and the minimum strain by a full load,

$$V = 0, \qquad \text{and } H = 48;$$

$$0 = z_1.4{,}27 + 48 \times 0{,}5 + 4 \times 12 + 8\,(9 + 6 + 3);$$

$$z_1\ (\text{min.}) = -50{,}6 \text{ tons};$$

and for z_2, separating line between Apexes 2 and 3,

$$V = 0{,}75, \qquad \text{and } H = 14{,}25;$$

$$0 = z_2.3{,}32 - 0{,}75 \times 9 + 14{,}25 \times 0{,}5 + 1 \times 9 + 2\,(6 + 3);$$

$$z_2\ (\text{max.}) = -8{,}25 \text{ tons};$$

and for the minimum, where

$$V = 0{,}75, \qquad \text{and } H = 45{,}75;$$

$$0 = z_2.3{,}32 + 0{,}75 \times 9 + 45{,}75 \times 0{,}5 + 4 \times 9 + 8\,(6 + 3);$$

$$z_2\ (\text{min.}) = -41{,}45 \text{ tons.}$$

fig. 188

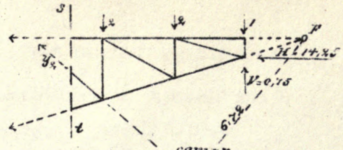


fig. 189

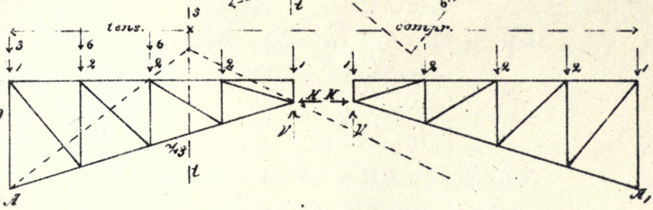


fig. 190

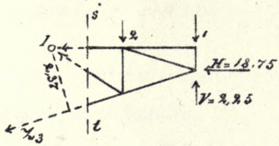


fig. 191

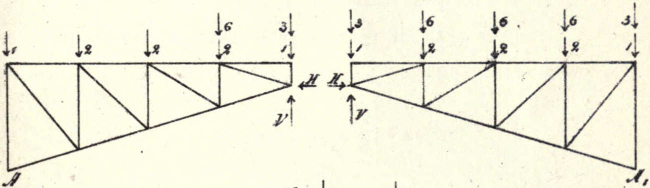


fig. 192

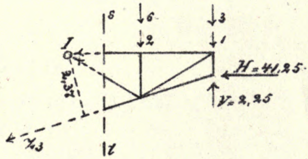


fig. 193

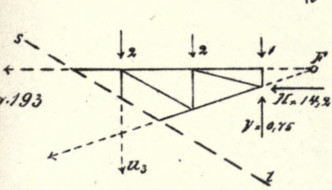


fig. 194

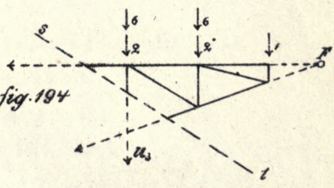
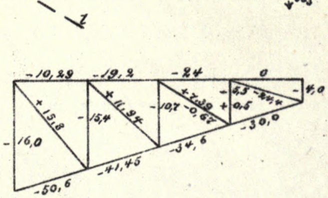


fig. 195



For z_4, separating the line between Apexes 4 and 5,

$$V = 4{,}5, \qquad \text{and } H = 25{,}5;$$

$$0 = z_4 . 1{,}423 - 4{,}5 \times 3 + 25{,}5 \times 0{,}5 + 1 \times 3;$$

$$z_4 \text{ (max.)} = -1{,}58 \text{ tons.}$$

For the minimum,

$$V = 4{,}5, \qquad \text{and } H = 34{,}5;$$

$$0 = z_4 . 1{,}423 + 4{,}5 \times 3 + 34{,}5 \times 0{,}5 + 4 \times 3;$$

$$z_4 \text{ (min.)} = -30{,}0 \text{ tons.}$$

Calculation of the Verticals u.

For u_3, per example, we have from Fig. 187,

$$V = 0{,}75, \qquad \text{and } H = 14{,}25;$$

193.] and the equation of equilibrium from Fig. 193,

$$0 = -u_3 . 7{,}5 + 0{,}75 \times 1{,}5 + 14{,}25 \times 0{,}5 - 1 \times 1{,}5 - 2\,(4{,}5 + 7{,}5) \text{ (rot. } r . F);$$

$$u_3 \text{ (max.)} = -2{,}3 \text{ tons;}$$

For the minimum from Fig. 185,

$$V = 3{,}75, \qquad \text{and } H = 23{,}25;$$

194.] and from Fig. 194,

$$0 = -u_3 . 7{,}5 + 3{,}75 \times 1{,}5 + 23{,}25 \times 0{,}5 - 1 \times 1{,}5 - 8\,(4{,}5 + 7{,}5) \text{ (rot } r . F);$$

$$u_3 \text{ (min.)} = -10{,}7 \text{ tons.}$$

In the same way we find for u_1, where $V = 0$, and $H = 48$,

$$u_1 \text{ (min.)} = -16{,}0 \text{ tons.}$$

For u_2, $\qquad V = 0, \qquad \text{and } H = 48;$

$$u_2 \text{ (min.)} = -15{,}4 \text{ tons;}$$

and for u_4, where $\quad V = 2{,}25, \qquad \text{and } H = 18{,}75;$

$$0 = -u_4 . 4{,}5 + 2{,}25 \times 1{,}5 + 18{,}75 \times 0{,}5 - 1 \times 1{,}5 - 2 . 4{,}5;$$

$$u_4 \text{ (max.)} = +0{,}5 \text{ tons.}$$

For the minimum,

$$V = 2{,}25, \qquad \text{and } H = 18{,}75;$$

$$0 = -u_4 . 4{,}5 + 2{,}25 \times 1{,}5 + 18{,}75 \times 0{,}5 - 1 \times 1{,}5 - 8 \times 4{,}5;$$

$$u_4 \text{ (min.)} = -5{,}5 \text{ tons};$$

$$u_5 = -4 \text{ tons}.$$

195.] The results are combined in Fig. 195.

II. *CALCULATION OF A TRUSS SUSTAINING A DOME.*

Plate 32, Figs. 196–7.] For the construction of a dome (Figs. 196–7), the distribution of the load at the apexes should be first considered. The ribs not being parallel, but intersecting at the vertex, therefore the variable load increases toward the base, AB, in the same relation as the sections of the horizontal circles drawn through the apexes.

These sections are proportional to their radii. When, therefore, their length is measured, and at a certain apex the load is defined, the proportion of the radius of this point compared with any other radius will give the weight or load at such apex. The weight of the rib itself will be the only constant load. Every other weight, as sheeting, snow and wind-pressure, should be calculated as a variable load—for instance, when the sheeting is displaced at one side for repairs.

This problem may be still further explained by the following example: On a dome of 100 yards span, the outlines may form a semi-globe whose radius = 51 yards. Its area being 16338 square yards, each of the eight ribs will sustain the weight of an area = 2042 square yards. The load, including the weight of sheeting, snow and wind-pressure, estimated at 470 lbs. per square yard, gives 480 tons (at 2000 lbs.) for each rib.

The weight of a rib whose outside and inside circles (chords) are 2 yards apart may be estimated = 60 tons, this being the permanent load, which when equally distributed to the panels, 15 in number, will give for each apex a permanent load of 4 tons.

For the proportion of the variable load, the distances of the

apexes from the vertical centre line require to be measured, resulting in

0	5,3	10,6	15,8	20,7	25,5	30	34,1	37,9	41,3	44,2	46,6	48,5	49,9	50,7	51
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15

Proportional to these figures is the variable load; therefore, when the whole sum (being 512) is divided into the whole load = 480 tons, and the result multiplied by those figures, then the variable load for each apex will be

5	9,9	14,8	19,4	23,9	28,1	32	35,5	38,7	41,4	43,7	45,5	46,8	47,6
1	2	3	4	5	6	7	8	9	10	11	12	13	14

This being defined, for the further calculation we suppose the ribs on their supports secured against the horizontal thrust by means of a horizontal wrought-iron band (ring); then the supports will sustain only a vertical pressure equal to the weight of the structure.

Each pair of opposite ribs thus fixed at the heels, and resting at the top against a globe or universal joint (providing expansion and contraction), will show conformity to the preceding example. In the calculation, therefore, we can follow the same principles.

So we find again by an unloaded and also by a completely loaded state the vertical pressure at the vertex = 0.

The horizontal force will be defined in forming the equation for all the moments of the loaded apexes in regard to the point A of support, their levers being

50	44,7	39,4	34,2	29,3	24,5	20	15,9	12,1	8,7	5,8	3,4	1,5	0,1	— 0,7
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14

For an unloaded state of ribs it will be

$$H.50 = 4\left(\tfrac{50}{2} + 44{,}7 + 39{,}4 + \ldots + 1{,}5 + 0{,}1 - 0{,}7\right) = 1056;$$

$$H = 21{,}12;$$

but for a loaded state,

$$H.50 = 4\left(\tfrac{50}{2} + 47{,}7 + 39{,}4 + \ldots + 1{,}5 + 0{,}1 - 0{,}7\right);$$

$$+ 5 \times 44{,}7 + 9{,}9 \times 39{,}4 + \ldots + 45{,}5 \times 1{,}5 + 46{,}8 \times 0{,}1) - 47{,}6 \times 0{,}7.$$

These figures show the moments of the movable load in regard to the point A, and for use in the following the value of each may be stated here:

223,5	390	506,2	568,4	585,6	562	508,8	429,6	336,7	240,1	148,6	68,3	4,7	—33,3
1	2	3	4	5	6	7	8	9	10	11	12	13	14

From this we find for H in the preceding equation,

$$H \times 50 = 1056 + 223{,}5 + 390 + \ldots + 68{,}3 + 4{,}7 - 33{,}3 = 5595;$$

$$H = 111{,}9.$$

Calculation of Strains x of the Outside Arch.

198.] To explain the calculation, we choose the section of the arch between the Apexes 5 and 6, M to be the point of rotation. (Fig. 198.)

The vertical line at E, distinguishing the influence of the load upon tensile and compressive strain, can be constructed in making a line from the opposite support through S, and another line from A through the point of rotation, M. The distance of this line from the vertical centre line is 13 yards, being between the Apexes 2 and 3.

The tension, x, will be a maximum when the points 3, 4, 5 . . . 14 are in an unloaded state, and the others in a loaded state; then for the pressure at the vertex for the formation of moments of both opposite ribs,

$$H.50 = 5595 - V \times 50 \text{ (for the rib to the right);}$$

$$H.50 = 5595 + V.50 - 506 - 568 - 586 - \ldots - 4{,}7 + 33{,}3$$
(for the rib to the left);

$$H = 72{,}7; \qquad V = 39{,}2.$$

Plate 33, Fig. 199.] From this, for x (max.), from Fig. 199,

$$0 = -x.2 - 72{,}7 \times 8{,}9 + 39{,}2 \times 26{,}7$$
$$+ 4\left(\tfrac{26{,}7}{2} + 21{,}4 + 16{,}1 + 10{,}9 + 6 + 1{,}2\right);$$
$$+ 5 \times 21{,}4 + 9{,}9 \times 16{,}1 \text{ (rot. } M);$$
$$x \text{ (max.)} = + 470{,}9 \text{ tons.}$$

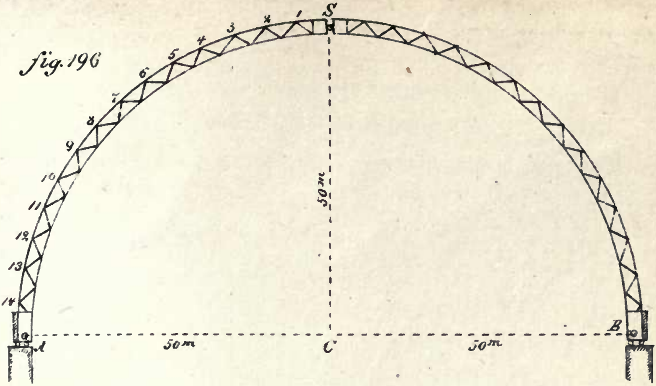


fig. 196

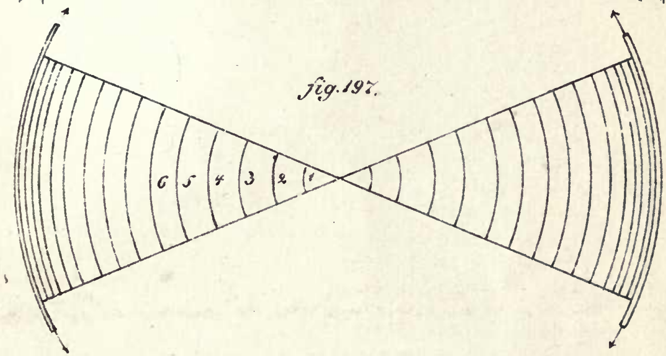


fig. 197.

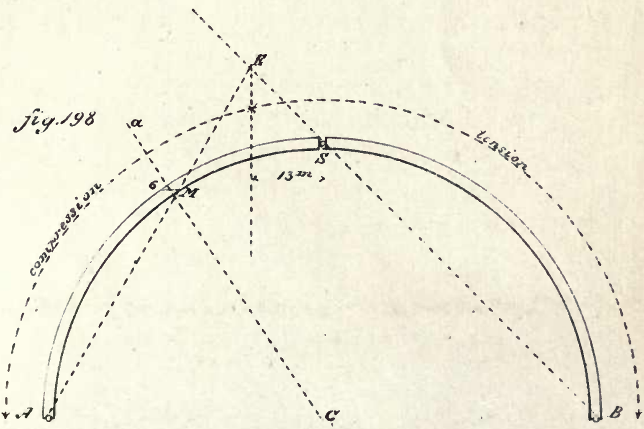


fig. 198

For x (min.) only the Apexes 3, 4, 5 ... 14 are supposed to be loaded; then the components of the resulting strain at the vertex of the ribs will be found in forming the equations for each of the two sides, adding for the left side to the permanent load the values of the variable load from Apexes 3 to 14.

$$0 = H \times 50 - V \times 50 - 1056;$$

$$0 = - H \times 50 - V . 50 + 1056 + 506 + 568 + 586 + \ldots + 4,7 - 33,3;$$

$$H = 60,3, \qquad \text{and } V = 39,2.$$

200.] Then for x (min.), from Fig. 200,

$$0 = - x . 2 - 60,3 \times 8,9 - 39,2 \times 26,7 + 4 \left(\tfrac{26,7}{2} + 21,4 + 16,1 + 10,9 + 6 + 1,2\right)$$

$$+ 14,8 \times 10,9 + 19,4 \times 6 + 23,9 \times 1,2;$$

$$x \text{ (min.)} = - 500,6 \text{ tons.}$$

Calculation of Strains z of the Inside Arch.

The same panel may serve for calculation, being the section opposite Apex 6.

For the equation of equilibrium of this section, the point of rotation will be in 6. To find the vertical line, distinguishing the

201.] action of the load upon tensile or compressive strain, the direction from A through 6 and from B through S gives the point of intersection, F (Fig. 201), being 17,3 yards from the vertical centre line between Apexes 3 and 4.

For z (max.), only the points 4, 5 ... 14 should be loaded; then, for the components at the vertex,

$$0 = H . 50 - V . 50 - 1056;$$

$$0 = H . 50 - V . 50 + 1056 + 568 + 586 + \ldots + 4,7 - 33,3;$$

$$H = 55,3, \qquad \text{and } V = 34,2.$$

202.] Now, from Fig. 202,

$$0 = z . 2 - 55,3 \times 8,74 - 34,2 \times 30;$$

$$+ 4 \left(\tfrac{30}{2} + 24,7 + 19,4 + 14,2 + 9,3 + 4,5\right)$$

$$+ 19{,}4 \times 9{,}3 + 23{,}9 \times 4{,}5;$$

$$z\ (\text{max.}) = + 436{,}5 \text{ tons};$$

and further, for the components at the vertex, for z (min.), the Apexes 4, 5 ... 14 are to be unloaded, all the others to be loaded; and for the negative value of the moments for Apexes 4 and 5, added to the sum of moments of the left rib, when entirely loaded,

$$0 = H.50 + V.50 - 5595;$$

$$0 = - H.50 + V.50 + 5595 - 568 - 586 - \ldots 4{,}7 + 33{,}3;$$

$$H = 77{,}7, \qquad \text{and } V = 34{,}2.$$

203.] Now, according to Fig. 203,

$$0 = z.2 - 77{,}7 \times 8{,}74 + 34{,}2 \times 30$$
$$+ 4\left(\tfrac{30}{2} + 24{,}7 + 19{,}4 + 14{,}2 + 9{,}3 + 4{,}5\right)$$
$$+ 5 \times 24{,}7 + 9{,}9 \times 19{,}4 + 14{,}8 \times 14{,}2;$$

$$z\ (\text{min.}) = - 610{,}5 \text{ tons.}$$

Calculation of the Diagonals y.

The diagonal between 9 and 10, intersecting with the outside arch at Apex 10, may serve for explanation.

Plate 34, Fig. 204.] In the cut st (Fig. 204), separating the diagonal and the arches, the point of intersection of the latter will be again in infinite distance (see calculation of trusses with parallel top and bottom flanges) in the tangent of a circle half way between the outside and inside arches.

This tangent forms an angle of 58½ degrees with the vertical line at the centre, or with the horizontal line CA.

The forces parallel to the tangent have no influence upon y, their direction going in infinite distance through the intersection of the inside and outside arches.

Therefore, all the forces acting upon the section Sst should be separated in forces right-angled and parallel to the tangent, the parallel forces to be omitted.

The vertical separating line of the weights acting upon compression or tension will be found in drawing a line through A (Fig. 204), parallel to the tangent, and through BS, intersecting at J. A

fig. 199

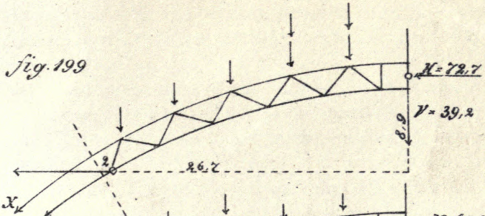


fig. 200

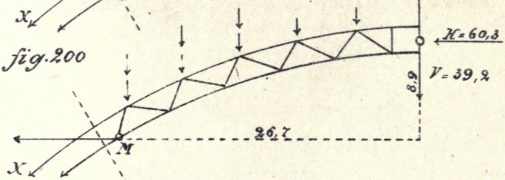


fig. 201.

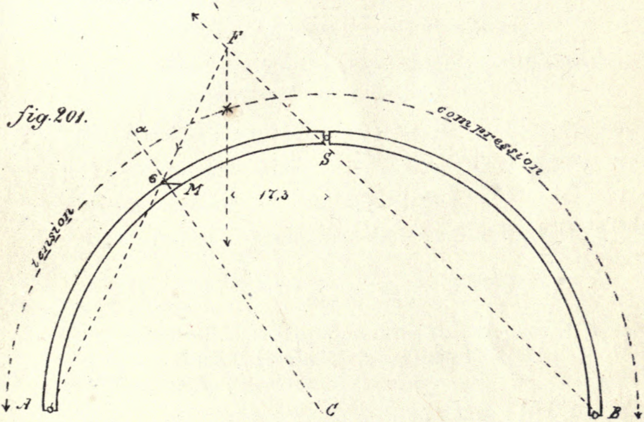


fig. 202

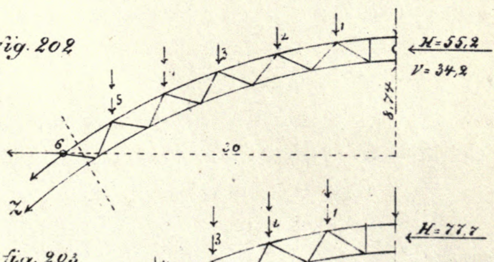
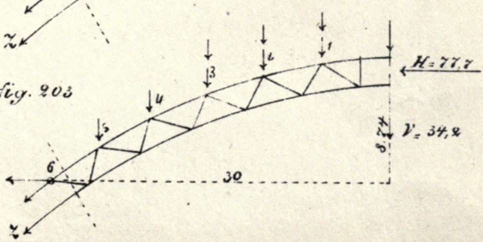


fig. 203



weight suspended in the vertical direction through J creates with its pressure at the vertex a resulting strain, being without influence upon the diagonal, y.

Every load to the right of the vertical line in J creates compression, and every load to the left to the separating line, st, produces tension. From this point down to the support, A, compression is produced.

The distance, J, from the vertex being 12 yards (*i.e.*, between the second and third apexes), the force, y, will be a maximum when the apexes 3, 4, 5, 6, 7, 8 and 9 are loaded.

For the forces at the vertex,

$$0 = H.50 - V.50 - 1056;$$

$$0 = -H.50 - V.50 + 1056 + 506 + 568 + 586 + 562 + 509 + 430 + 337;$$

$$H = 56{,}1 \text{ tons}, \qquad \text{and } V = 35 \text{ tons}.$$

205.] From Fig. 205, for the equilibrium, all the forces acting in the direction N being 0,

$$0 = N + 56{,}1 \times \cos 31\tfrac{1}{2}° + 35 \times \sin 31\tfrac{1}{2}° - 4 \times 9{,}5 \times \sin 31\tfrac{1}{2}°$$
$$- (14{,}8 + 19{,}4 + 23{,}9 + 28{,}1 + 32 + 35{,}5 + 38{,}7) \sin 31\tfrac{1}{2}°$$

the solution being $\qquad N \text{ (max.)} = 54{,}3,$

which gives for y, forming with the force N an angle of 52° 35′,

$$y \text{ (max.)} = \frac{54{,}3}{\cos 52° 35'} = +\ 89{,}3 \text{ tons}.$$

y (min.) will be defined when, with the exception of Apexes 3, 4, 5, 6, 7, 8 and 9, all the others are loaded.

Then, for the pressure at the vertex,

$$0 = H.50 + V.50 - 5595;$$

$$0 = -H.50 + V.50 + 5595 - 506 - 568 - 586 - 562 - 509 - 430 - 337;$$

$$H = 77, \qquad \text{and } V = 35;$$

206.] and from Fig. 206, for N (min.),

$$0 = N + 77 \cos 31\tfrac{1}{2}° - 35 \sin 31\tfrac{1}{2}° - 4 \times 9{,}5 \times \sin 31\tfrac{1}{2}°$$
$$- (5 + 9{,}9) \sin 31\tfrac{1}{2}°;$$

$$N\ (\text{min.}) = -19{,}7\,;$$

therefore, for y, $$y = \frac{-19{,}7}{\cos 52^\circ\ 35'} = -32{,}5 \text{ tons.}$$

This may be sufficient to show the calculation of the single members, and we still want to show only the calculation of the strain, S, in the horizontal ring or band at the heels of the ribs.

207.] As in the case above described there are only a limited number of ribs, forming at their base a polygon (Fig. 207), which will also be the form of the ring, then

$$2 \sin 22\tfrac{1}{2}^\circ = H\,;$$

$$S = \frac{111{,}9}{2\,.\,0{,}3827} = 146{,}1 \text{ tons.}$$

But when numerous ribs form the dome, so that the horizontal

208.] thrust toward the ring (as in Fig. 208) is equally distributed, and p the horizontal pressure per unit of length, then

$$p \times 2\,r\varphi = 2\,S\varphi, \qquad \text{or } S = p\,.\,r.$$

So for our example,

$$p = \frac{H}{r\cdot\frac{\pi}{4}} = \frac{111{,}9}{51 \times 0{,}785} = 2{,}794\,;$$

and therefore $$S = 2{,}794 \times 51 = 142{,}5 \text{ tons.}$$

[Plates 29, 30, 31, 32, 33 and 34—embracing Figs. 179 to 208.]

THE END.

fig. 204

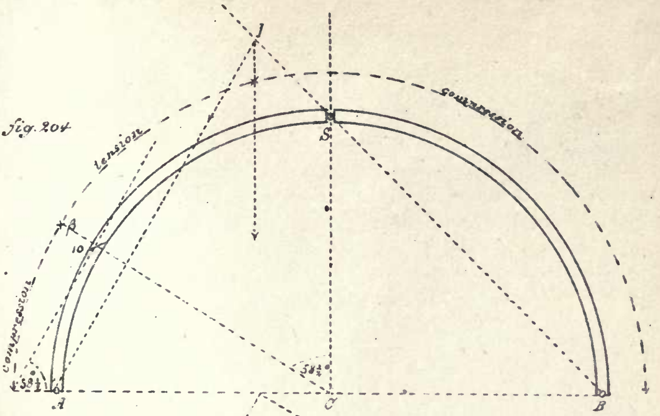


fig. 205

fig. 206

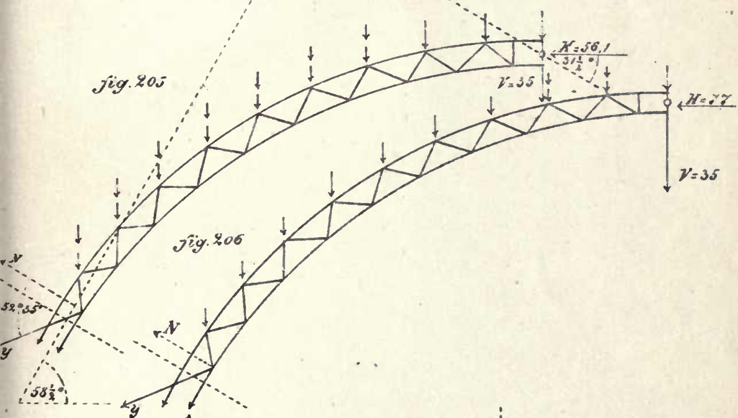


fig. 207

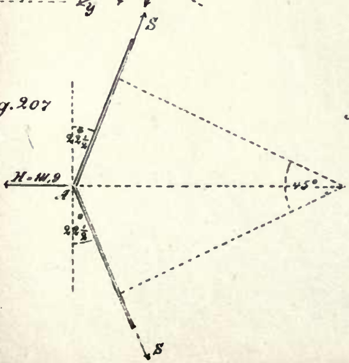


fig. 208